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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.
During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.
Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM
Set 193

JEE MAIN

1. A picnic is to be held on a particular day. The weather forecast says that there is 80% chance of rain on that day. If it rains, the probability of a good picnic is 0.3, and if it does not, the probability is 0.9. The probability that the picnic will be good is

(a) 0.8 (b) 0.3 (c) 0.9 (d) 0.42

2. If equation $(a-2)(x-[x])^2+2(x-[x])+a^2=0$ (where $[x]$ denotes G.I.F and $a \in R$) has no integral solution and has exactly one solution in (2, 3), then a lies in the interval

(a) (0, 1) (b) (1, 2) (c) (−1, 3) (d) (−1, 0)

3. If $f(x)$ is a continuous function satisfying $f(x)\,f(1/x)=f(x)+f(1/x)$ and $f(1)>0$, then $\lim\limits_{x\to 1} f(x)$ is equal to

(a) 2 (b) 1 (c) 3 (d) 5

4. In the given coordinate plane, the region M consists of all the points (x, y) satisfying the inequalities $y \le x$, $y \ge 0$ and $y \le 2-x$ simultaneously. The region N which varies with parameter 't', consists all the points (x, y) satisfying $t \le x \le t+1$ and $0 \le t \le 1$ and simultaneously area of $M \cap N$ is

(a) $-t^2+t+\frac{1}{2}$ (b) $-2t^2+2t$

(c) $1-\frac{t^2}{2}$ (d) $\frac{(t-1)^2}{2}$

5. If two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points, then

(a) $2<r<8$ (b) $r<2$

(c) $r>2$ (d) $r=2$

JEE ADVANCED

6. Suppose that a, b, c are real numbers such that $a+b+c=1$. If the matrix $A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ be an orthogonal matrix, then

(a) A is an involutory matrix

(b) $|A|$ is negative

(c) $a^3+b^3+c^3-3abc=1$

(d) atleast one of a, b, c is negative

COMPREHENSION

Two arithmetic progressions have the same number of terms. The ratio of the last term of the first progression to the first term of the second progression is equal to the ratio of the last term of the second progression to the first term of the first progression and is equal to 4. The ratio of the sum of the n terms of the first progression to the sum of n terms of the second progression is equal to 2.

7. Ratio of the first term of the first progression to the first term of the second progression is

(a) 8 (b) 2/7 (c) 26 (d) 7/2

8. Ratio of their common differences is

(a) 26 (b) 7/2 (c) 2 (d) 8

INTEGER TYPE

9. If $iz^3+z^2-z+i=0$, then $[|z|]=$
where [.] is G.I.F.

MATRIX MATCH

10. Match the following:

Column-I		Column-II	
P.	Maximum of $4\sin^2 x+3\cos^2 x+\sin\frac{x}{2}+\cos\frac{x}{2}-\sqrt{2}$ is	1.	10
Q.	Minimum of $\cos 2\theta+\cos\theta+\left(\frac{9}{8}\right)$ is	2.	4
R.	Maximum of $\lvert 3\sin x+4\cos x-5\rvert$ is	3.	0

	P	Q	R
(a)	3	1	2
(b)	1	3	2
(c)	2	3	1
(d)	1	2	3

See Solution Set of Maths Musing 192 on page no. 85

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(for JEE / Olympiad Aspirants)

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More focus on two subjects can improve my rank in JEE	**JEE paper contains basic & conceptual questions. Skipping one subject can reduce your marks which could result in low ranking and not getting cutoff marks.** Suggestion - Focus on all subjects (PCM) & try to solve basic and conceptual questions of all topics.

Please visit youtube to watch more myth and reality discussions on KCS EDUCATE channel.

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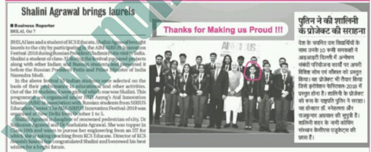
Knowledge Quiz - 14

Find all 4-tuples (a, b, c, d) of natural numbers with $a \le b \le c$ and $a! + b! + c! = 3^d$.

Please send your detailed solution before 15th January 2019 to **quiz@kcseducate.in** along with your name, father's name, class, school, address and contact number.

Winner Knowledge Quiz - 13

- **Nikhil Narula, (Class - X), D. C. Model Senior Secondary School, Ferozepur, Punjab.**
- **Soumyajoy Kundu, (Class - XI), St.Xavier's Institution, Kolkata, West Bengal.**
- **Rohit Mandhyan, (Class - XII), Bakhshi's Springdales Sr. Sec. School, Kota, Rajasthan.**



Shalini Agrawal brings laurels

Business Reporter
BHILAI, Oct 7

BHILAI lass and a student of KCS Educate, Shalini Agrawal brought laurels to the city by participating in the AIM-SIRIUS Innovation Festival-2018 during Russian President Vladimir Putin visit to India. Shalini a student of class-XI during the festival prepared projects along with other Indian and Russian students and presented it before the Russian President Putin and Prime Minister of India Narendra Modi.

In the above festival 10 Indian students were selected on the basis of their performance in educational and other activities. Out of the 10 students 5 were girls of which one was Shalini. This programme was organised under NITI Aayog's Atal Innovation Mission (AIM) in association with Russian students from SIRIUS Education Center. The AIM-SIRIUS Innovation Festival-2018 was organised at New Delhi from October 1 to 5.

Shalini Agrawal is daughter of renowned pedestrian of city, Dr Rajkumar Agrawal and Dr Snehalata Agrawal. She was topper in Class-10th and wants to pursue her engineering from an IIT for which she is taking coaching from KCS Educate. Director of KCS Avanish Saxena has congratulated Shalini and bestowed his best wishes for a bright future.

Thanks for Making us Proud !!!

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CONCEPT BOOSTERS

Parabola

This column is aimed at Class XI students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

***ALOK KUMAR**

INTRODUCTION

- A conic section or conic is the locus of a point, which moves such that its distance from a fixed point is in a constant ratio to its distance from a fixed straight line, not passing through the fixed point. The fixed point is called the focus. The fixed straight line is called the directrix. The constant ratio is called eccentricity which is denoted by e.
- When $e = 1$, the conic is a parabola. When $e < 1$, the conic is an ellipse. When $e > 1$, the conic is a hyperbola.
- The line of symmetry of the conic section is called its axis.
- A point of intersection of a conic with its axis is called vertex.
- A chord passes through the focus and perpendicular to the axis is called latus rectum.

GENERAL EQUATION OF CONIC

- Let $S(\alpha, \beta)$ be the focus and $Ax + By + C = 0$ be the equation of the directrix QN.
Let $P(x, y)$ be any point on it and let $PN \perp Q$, then by definition

$$\frac{SP}{PN} = e \Rightarrow SP^2 = e^2.PN^2$$

$$\Rightarrow (x - \alpha)^2 + (y - \beta)^2 = e^2\left(\frac{Ax + By + C}{\sqrt{A^2 + B^2}}\right)^2$$

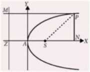
which on simplification takes the form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0,$$

which is the general equation of second degree. This will represent

(i) a pair of straight lines if $\Delta = 0$, where $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$, $h^2 \geq ab$
(ii) a circle if $\Delta \neq 0$, $h = 0$, $a = b$
(iii) a parabola if $\Delta \neq 0$, $ab - h^2 = 0$
(iv) an ellipse if $\Delta \neq 0$, $ab - h^2 > 0$
(v) a hyperbola if $\Delta \neq 0$, $ab - h^2 < 0$

STANDARD EQUATION OF A PARABOLA



- Let P be the moving point and A be the origin. Here SZ is the axis of the parabola. Now, the middle point of SZ, say A, will lie on the locus of P *i.e.*, $AS = AZ$.

Let $AS = a$, then $ZA = a$. Let (x, y) be the coordinates of point P.

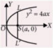
$\therefore$ $MP = ZN = ZA + AN = a + x$. Also, $MP = PS$

$\Rightarrow$ $MP^2 = PS^2 \Rightarrow (a + x)^2 = (x - a)^2 + y^2$.

Hence, the equation of parabola is $y^2 = 4ax$.

Latus Rectum of the Parabola

- Any chord of the parabola $y^2 = 4ax$ which is ⊥ to its axis is called the double ordinate.



- The given parabola be $y^2 = 4ax$.
In the figure LSL' is the latus rectum.
Also, $LSL' = 2(\sqrt{4a.a})$
$= 4a =$ double ordinate through the focus S.

---

* Alok Kumar, a B Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

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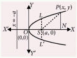
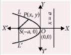
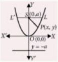
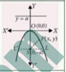
Different Forms of The Parabola

Form	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Shape of the parabola	 Y P(x, y) L x = −a X′ O (0,0) S (a, 0) N X L′ Y′	 Y L P(x, y) x = a X′ S(−a, 0) O (0,0) X L′ Y′	 Y L′ S(0,a) L P(x, y) X′ O (0,0) X y = −a Y′	 Y y = a O (0,0) X′ X P(x, y) L′ S(0, −a) L Y′
Vertex	(0, 0)	(0, 0)	(0, 0)	(0, 0)
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Tangent at the vertex	$x = 0$	$x = 0$	$y = 0$	$y = 0$
Parametric Equation	$x = at^2, y = 2at$	$x = -at^2, y = 2at$	$x = 2at, y = at^2$	$x = 2at, y = -at^2$
Focal Distance of (x, y)	$x + a$	$-x + a$	$y + a$	$-y + a$

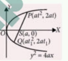
Equation of Tangent

At the point (x_1, y_1)	$yy_1 = 2a(x + x_1)$	$yy_1 = -2a(x + x_1)$	$xx_1 = 2a(y + y_1)$	$xx_1 = -2a(y + y_1)$
In terms of m	$y = mx + a/m$	$y = mx - a/m$	$x = my + a/m$	$x = my - a/m$
In terms of t	$yt = x + at^2$	$yt = -x + at^2$	$xt = y + at^2$	$xt = -y + at^2$

Equation of Normal

At the point (x_1, y_1)	$y - y_1 = \frac{-y_1}{2a}(x - x_1)$	$y - y_1 = \frac{y_1}{2a}(x - x_1)$	$x - x_1 = \frac{-x_1}{2a}(y - y_1)$	$x - x_1 = \frac{x_1}{2a}(y - y_1)$
In terms of m	$y = mx - 2am - am^3$	$y = mx + 2am + am^3$	$x = my - 2am - am^3$	$x = my + 2am + am^3$
In terms of t	$y = -tx + 2at + at^3$	$y = tx + 2at + at^3$	$x = -ty + 2at + at^3$	$x = ty + 2at + at^3$

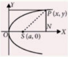
Focal Chord



Any chord to the parabola $y^2 = 4ax$ which passes through the focus is called a focal chord of the parabola $y^2 = 4ax$.

If the equation of chord PQ passes through $(a, 0)$, then $t_1 t = -1$.

Focal Distance of Any Point



The distance between the point P on the conic and the focus S is called focal distance *i.e.*, PS.

$\therefore$ Focal distance $= PS = a + x$

Position of a Point Relative to the Parabola

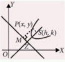
Consider the parabola $y^2 = 4ax$ and $P(x_1, y_1)$ be a point.

If $y_1^2 - 4ax_1 = 0$, then the point P lies on the parabola.

If $y_1^2 - 4ax_1 > 0$, then P lies outside the parabola.

If $y_1^2 - 4ax_1 < 0$, then P lies inside the parabola.

GENERAL EQUATION OF A PARABOLA



Let (h, k) be the focus S and $lx + my + n = 0$ the equation of the directrix ZM of a parabola. Let (x, y) be the coordinates of any point P on the parabola. Then the relation,

PS = distance of P from ZM, gives

$$(x - h)^2 + (y - k)^2 = (lx + my + n)^2/(l^2 + m^2)$$

$$\Rightarrow \quad (mx - ly)^2 + 2gx + 2fy + d = 0$$

which is the general equation of a parabola.

INTERSECTION OF A STRAIGHT LINE WITH THE PARABOLA

Points of intersection of parabola $y^2 = 4ax$ and line $y = mx + c$ are given by $(mx + c)^2 = 4ax$

i.e., $m^2x^2 + 2x(mc - 2a) + c^2 = 0$ ...(i)

Since (i) is a quadratic equation, the straight line meets the parabola in two points (real, coincident,

MtG

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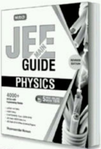

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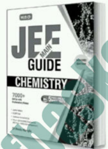

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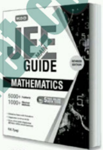

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or imaginary). The roots of (i) are real or imaginary according as $\{2(mc - 2a)\}^2 - 4m^2c^2$ is positive or negative, *i.e.* according as $-amc + a^2$ is positive or negative, *i.e.* according as mc is less than or greater than a.

- When $m = 0$, eqn. (i) gives $x = \dfrac{c^2}{4a}$ *i.e.*, only one value of x. So, line parallel to the axis of the parabola meets the parabola only in one point.

LENGTH OF THE CHORD

The abscissae of the points common to the straight line $y = mx + c$ and the parabola $y^2 = 4ax$ are given by the equation $m^2x^2 + (2mc - 4a)\,x + c^2 = 0$.
If (x_1, y_1) and (x_2, y_2) are the points of intersection, then
$$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2 = \frac{16a(a - mc)}{m^4}$$
and $(y_1 - y_2) = m(x_1 - x_2)$

Hence, the required length $= \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$
$$= \sqrt{1 + m^2}\,|(x_1 - x_2)| = \frac{4}{m^2}\sqrt{1 + m^2}\,\sqrt{a(a - mc)}$$

EQUATION OF THE TANGENTS FROM AN EXTERNAL POINT

Let $y^2 = 4ax$ be the parabola and $P(x_1, y_1)$ be an external point. Then, equations of the tangents from (x_1, y_1) to the given parabola are given by
$SS_1 = T^2$, where $S = y^2 - 4ax$, $S_1 = y_1^2 - 4ax_1$, $T = yy_1 - 2a(x + x_1)$

Note : (1) The point of intersection of the tangents at 't_1' and 't_2' to the parabola $y^2 = 4ax$ is $(at_1t_2, a(t_1 + t_2))$.

(2) For the parabola, $y^2 = 4ax$ the point of contact of the tangent $y = mx + c$, where $c = \dfrac{a}{m}$ is given by $\left(\dfrac{a}{m^2}, \dfrac{2a}{m}\right)$.

CHORD OF CONTACT

Equation of the chord of contact of the tangents drawn from a point (x_1, y_1) to the parabola $y^2 = 4ax$ is $T = 0$, *i.e.* $yy_1 - 2a(x + x_1) = 0$.

Equation of a Chord with Midpoint (x_1, y_1)

The equation of the chord of the parabola $y^2 = 4ax$ with mid point (x_1, y_1) is $T = S_1$
i.e., $yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$

Co-Normal Points

For the parabola $y^2 = 4ax$ and a point $P(x_1, y_1)$equation of normal is given by $y = mx - 2am - am^3$.
or $am^3 + m(2a - x_1) + y_1 = 0$, which is cubic in m and has three roots say m_1, m_2, m_3.

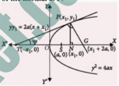
(i) $m_1 + m_2 + m_3 = 0$
(ii) $m_1m_2 + m_2m_3 + m_3m_1 = (2a - x_1)/a$
(iii) $m_1m_2m_3 = -y_1/a$

Note: (1) If normal at the point t_1 meets the parabola again at t_2, then $t_2 = -t_1 - 2/t_1$.

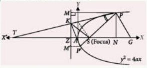
(2) Point of intersection of the normals to the parabola $y^2 = 4ax$ at $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is $(2a + a(t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1 + t_2))$.

Subtangent and Subnormal

Let the tangent and normal at any point $P(x_1, y_1)$ on the parabola $y^2 = 4ax$ meet the axis in T and G respectively. Then PT is called the length of the tangent at P and PG is called the length of the normal at P.
NT is subtangent and NG is subnormal at P. The coordinates of T and G can be easily found by putting $x = 0$ in the equations of the tangent and normal at P. It is evident from figure that
Subtangent $= NT =$ twice the abscissa of point P
Subnormal $= NG = 2a =$ semi latus rectum



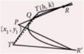
PROPERTIES OF THE PARABOLA

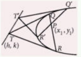


- The tangent at any point P of a parabola bisects the angle between the focal chord through P and the perpendicular from P on the directrix.
- The portion of a tangent to a parabola cut off between the directrix and the curve subtends a right angle at the focus.
- Tangents at the extremities of a focal chord intersect at right angles on the directrix.
- Any tangent to a parabola and the perpendicular on it from the focus meet on the tangent at the vertex.

Pole and Polar

Let Q and R be the points in which any chord drawn through the point P, whose coordinates are (x_1, y_1), meets the parabola.
Let the tangents at Q and R meet in the point whose coordinates are (h, k).





We require the locus of (h, k).

Since QR is the chord of contact of tangents from (h, k), so its equation is $ky = 2a\,(x + h)$

Since this straight line passes through the point (x_1, y_1). $\therefore \quad ky_1 = 2a\,(x_1 + h)$

It follows that the point (h, k) always lies on the straight line $yy_1 = 2a(x + x_1)$

PROBLEMS

Single Correct Answer Type

1. If (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then its equation is
(a) $x^2 + 8y = 32$ (b) $y^2 + 8x = 32$
(c) $x^2 - 8y = 32$ (d) $y^2 - 8x = 32$

2. Co-ordinates of the focus of the parabola $x^2 - 4x - 8y - 4 = 0$ are
(a) (0, 2) (b) (2, 1) (c) (1, 2) (d) (–2, –1)

3. If $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are two variable points on the curve $y^2 = 4ax$ and PQ subtends a right angle at the vertex, then t_1t_2 is equal to
(a) –1 (b) –2 (c) –3 (d) –4

4. In the parabola $y^2 = 4ax$, the length of the chord passing through the vertex and inclined to the axis at an angle $\pi/4$ is
(a) $4a\sqrt{2}$ (b) $\frac{4a}{\sqrt{2}}$
(c) $2a\sqrt{2}$ (d) none of these

5. The coordinates of a point on the parabola $y^2 = 8x$, whose focal distance is 4, are
(a) $\left(\frac{1}{2}, \pm 2\right)$ (b) $(1, \pm 2\sqrt{2})$
(c) $(2, \pm 4)$ (d) none of these

6. Two tangents are drawn from the point (–2, –1) to the parabola $y^2 = 4x$. If α is the angle between these tangents, then $\tan\alpha$ equals to
(a) 3 (b) 1/3 (c) 2 (d) 1/2

7. The angle between tangents to the parabola $y^2 = 4ax$ at the point where it intersects with the line $x - y - a = 0$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

8. The focus of the parabola $x^2 - 8x + 2y + 7 = 0$ is
(a) $\left(0, -\frac{1}{2}\right)$ (b) $\left(4, \frac{9}{2}\right)$
(c) (4, 4) (d) $\left(-4, -\frac{9}{2}\right)$

9. For the parabola $y^2 = 4ax$, the ratio of the subtangent to the abscissa is
(a) 1 : 1 (b) 2 : 1 (c) $x : y$ (d) $x^2 : y^2$

10. The length of subnormal to the parabola $y^2 = 4ax$ at any point is equal to
(a) $a\sqrt{2}$ (b) $2\sqrt{2}a$ (c) $a/\sqrt{2}$ (d) $2a$

11. The length of the latus rectum of the parabola $x = ay^2 + by + c$ is
(a) $\frac{a}{4}$ (b) $\frac{a}{3}$ (c) $\frac{1}{a}$ (d) $\frac{1}{4a}$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

					3		2	5					1	2	5		7			6
						1											8			7
1				8	5								7						5	2
			6				3	9								8	3			
		5				6								4				9		1
	4			2									5					7		
	3					2	4							7	2	9		3		
			9	1				6		4				9	7		6			5
	7	8		3					6		8									
									3	5		8								
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2		3											2				1			6
5	1					4	9	3	8		5		7		9	4				
	2		4	5	8									6			9			
			6									5	4		8				1	
8	7			9										9		7		4		5
								1										7	5	
		2	7				3	4					9			8			2	
				1			5	8							2					4

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

12. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum is
(a) 12 sq. units (b) 16 sq. units
(c) 18 sq. units (d) 24 sq. units

13. If the line $x + y - 1 = 0$ touches the parabola $y^2 = kx$, then the value of k is
(a) 4 (b) –4 (c) 2 (d) –2

14. The coordinates of the point on the parabola $y = x^2 + 7x + 2$, which is nearest to the straight line $y = 3x - 3$ are
(a) (–2, –8) (b) (1, 10) (c) (2, 20) (d) (–1, –4)

15. The point (1, 2) is one extremity of focal chord of parabola $y^2 = 4x$. The length of this focal chord is
(a) 2 (b) 4
(c) 6 (d) none of these

16. If the straight line $y = mx + c$ $(m > 0)$ touches the parabola $y^2 = 8(x + 2)$, then the minimum value taken by c is
(a) 12 (b) 8 (c) 4 (d) 6

17. The length of the subtangent to the parabola $y^2 = 16x$ at the point whose abscissa is 4, is
(a) 2 (b) 4
(c) 8 (d) none of these

18. If the line $x + y = 1$ touches the parabola $y^2 - y + x = 0$, then the co-ordinates of the point of contact are
(a) (1, 1) (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (c) (0, 1) (d) (1, 0)

19. If (x_1, y_1) and (x_2, y_2) are the ends of a focal chord of the parabola $y^2 = 4ax$, then $x_1x_2 + y_1y_2 =$
(a) $-3a^2$ (b) $3a^2$ (c) $-4a^2$ (d) $4a^2$

20. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at the vertex, then
(a) $4am + n = 0$ (b) $4am + 4al + n = 0$
(c) $4al + n = 0$ (d) $al + n = 0$

21. Equation of parabola having it's vertex at $A(1,0)$ and focus at $S(3, 0)$ is
(a) $y^2 = 4(x - 1)$ (b) $y^2 = 4(x + 1)$
(c) $y^2 = 8(x - 1)$ (d) $y^2 = 8(x + 1)$

22. Equation of parabola having the extremities of it's latus rectum as (3, 4) and (4, 3) is

(a) $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 5)^2}{2}$

(b) $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 8)^2}{2}$

(c) $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 4)^2}{2}$

(d) None of these

23. Locus of mid-point of chords of the parabola $y^2 = 4ax$ that pass through the point $(3a, a)$ is
(a) $y^2 + 2ax + ay - 6a^2 = 0$ (b) $y^2 + 2ax - ay + 6a^2 = 0$
(c) $y^2 - 2ax + ay + 6a^2 = 0$ (d) $y^2 - 2ax - ay + 6a^2 = 0$

24. If the line joining the points $A(at_1^2, at_1)$ and $B(at_2^2, at_2)$ passes through $C(0, b)$, then
(a) $b(t_1 + t_2) = 2at_1t_2$ (b) $2b(t_1 + t_2) = at_1t_2$
(c) $b(t_1 + t_2) = at_1t_2$ (d) None of these

25. If the lines $(y - b) = m_1(x + a)$ and $(y - b) = m_2(x + a)$ are the tangents of $y^2 = 4ax$, then
(a) $m_1 + m_2 = 0$ (b) $m_1m_2 = 1$
(c) $m_1m_2 = -1$ (d) $m_1 + m_2 = 1$

26. Tangents drawn to parabola $y^2 = 4ax$ at the points A and B intersect at C. Ordinate of A, C and B are in
(a) A.P. (b) G.P.
(c) H.P. (d) None of these

27. Tangents drawn to parabola $y^2 = 4ax$ at the point A and B intersect at C. If 'S' be the focus of the parabola then, SA, SC and SB are in
(a) A.P. (b) G.P.
(c) H.P. (d) None of these

28. Locus of trisection point of any arbitrary double ordinate on the parabola $x^2 = 4by$ is
(a) $9x^2 = by$ (b) $3x^2 = 2by$
(c) $9x^2 = 4by$ (d) $9x^2 = 2by$

29. If three distinct and real normals can be drawn to $y^2 = 8x$ from the point $(a, 0)$, then
(a) $a > 2$ (b) $a > 4$
(c) $a \in (2, 4)$ (d) None of these

30. The parabola $y^2 = 4x$ and the circle $(x - 6)^2 + y^2 = r^2$ will have no common tangent if 'r' is equal to
(a) $r > \sqrt{20}$ (b) $r < \sqrt{20}$
(c) $r > \sqrt{18}$ (d) $r \in (\sqrt{20}, \sqrt{28})$

31. Parabola $y^2 = 4x$ and the circle having it's centre at (6, 5) intersect at right angle. Possible point of intersection of these curves can be
(a) (9, 6) (b) $(2, \sqrt{8})$ (c) (1, 2) (d) $(3, 2\sqrt{3})$

32. Normal *PO*, *PA* and *PB* ('*O*' being the origin) are drawn to $y^2 = 4x$ from $P(h, 0)$. If $\angle AOB = \pi/2$, then area of quadrilateral *OAPB* is equal to

(a) 12 sq. units (b) 24 sq. units
(c) 6 sq. units (d) 18 sq. units

33. Radius of the circle that passes through origin and touches the parabola $y^2 = 4ax$ at the point $(a, 2a)$ is

(a) $\frac{5}{\sqrt{2}}a$ (b) $2\sqrt{2}a$ (c) $5\sqrt{2}a$ (d) $3\sqrt{2}a$

34. If the line $y = mx + c$ is a tangent to $y^2 = 4mx$, then distance of this tangent from the parallel normal is

(a) $\sqrt{1+m^2}$ (b) $\sqrt{2+m^2}$
(c) $(1 + m^2)^{3/2}$ (d) $(2 + m^2)^{3/2}$

35. Angle between the tangents drawn to $y^2 = 4x$ at the points where it is intersected by the line $y = x - 1$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

36. Maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ can be equal to

(a) 3 (b) 4 (c) 6 (d) 5

37. If two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4by$, then

(a) $|b| > \frac{1}{2\sqrt{2}}$ (b) $|b| < \frac{1}{2\sqrt{2}}$
(c) $|b| > \frac{1}{\sqrt{2}}$ (d) $|b| < \frac{1}{\sqrt{2}}$

Multiple Correct Answer Type

38. A circle of radius '*r*' touches the parabola $y^2 + 4x = 0$ at the vertex of the parabola. The centre of the circle lies to the left of the vertex and this circle lies completely within the parabola then exhaustive range of '*r*' belongs to

(a) $\left(1, \frac{3}{2}\right]$ (b) $(0, 2)$
(c) $\left(0, \frac{5}{2}\right]$ (d) $(0, 3)$

39. If $f(x + y) = f(x).f(y)$ for all *x*, *y* and $f(1) = 2$ and $a_r = f(r)$, for $r \in N$, then the coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4, may be

(a) (a_1, a_2) (b) $(a_1, -a_2)$ (c) (a_1, a_1) (d) (a_2, a_2)

IISc, IIT-Delhi in Top 100 of Global University Employability Rankings

The Indian Institute of Science (IISc) in Bangalore, Indian Institute of Technology (IIT) Delhi and Bombay are among the top education institutions from the country that have made it to the top 150 list of the 'Global University Employability Ranking'.

The annual ranking is produced by French HR consultancy Emerging and published by the 'Times Higher Education', lists the top 150 institutions worldwide for employability, based on a global survey of around 7,000 recruitment and international managers from major businesses. The Harvard University topped the list, followed by the California Institute of Technology and Massachusetts Institute of Technology.

"India's performance in the employability ranking has been somewhat inconsistent since the list was first launched in 2011. It still only counts three universities in the top 150, suggesting it's struggling to make the reputational ground with employers that some other Asian nations have achieved," said Simon Baker, Data Editor at 'Times Higher Education'.

The ranking shows that while India has seen some improvement to its representation and institutional positions in the table since 2011, it has not kept pace with advances made by other countries in the region and globally. "There is some solid improvement this year for India, however. In the 2018 global listing, the Indian Institute of Science rises one position to 28th, while the Indian Institute of Technology Delhi soars this year to place global 53rd, from 145 last year," notes the report.

There is a noticeable difference in the emphasis placed on graduates having critical thinking skills between countries in East Asia and Europe or North America. In the US, 90 per cent of employers rated critical thinking skills as very important, while in China this fell to 75 per cent. The US continues to dominate the ranking as the data analysis reveals a swiftly narrowing global employability gap. America has 34 institutions in the top 150 this year, compared to 55 in 2011, with six universities in the top 10, a fall from seven last year.

The UK largely holds its position in this year's ranking – retaining 10 institutions. But US and UK universities are also found to be struggling to keep pace with global competitors in preparing students for the modern workplace. The new data analysis reveals a substantial global shift in graduate employability this decade. We see a dramatically improved performance within East Asia and parts of Europe is visible. By-and-large, the highest risers are those equipping students with softer skills increasingly favoured among recruiters, such as teamwork, combined with the strongest possible industry experience.

To produce the 2018 'Global University Employability Ranking', an online survey was completed by two panels of participants between May and September 2018. The panels included respondents from 22 countries: Argentina, Australia, Brazil, Canada, China, France, Germany, India, Israel, Italy, Japan, Mexico, Morocco, the Netherlands, Singapore, South Africa, South Korea, Spain, the United Arab Emirates, Sweden, the UK and the US.

◆◆

40. If the parabolas $y^2 = 4kx(k > 0)$ and $y^2 = 4(x - 1)$ do not have a common normal other than the axis of parabola, then $k \in$

(a) $(0, 1)$ (b) $(2, \infty)$ (c) $(3, \infty)$ (d) $(0, \infty)$

41. The points on axis of parabola $3x^2 + 4x - 6y + 8 = 0$ from which three distinct normals can be drawn to it are

(a) $\left(\frac{-2}{3}, 2\right)$ (b) $\left(\frac{-2}{3}, 3\right)$

(c) $\left(\frac{-2}{3}, 4\right)$ (d) $\left(\frac{-2}{3}, 1\right)$

Assertion & Reason Type

***Directions :** In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :*

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

42. Statement-1 : The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h, k), then centroid of ΔPQR lies on $x = 0$.

Statement-2 : If A, B, C are three co-normal points of the parabola $y^2 = 4ax$, then centroid of ΔABC lies on axis of the parabola.

43. Statement-1 : Let 'a' and 'b' be the segments of the focal chord and $2l$ be its latus rectum, then $a^3 + b^3 > 2l^3$.

Statement-2 : A.M. > G.M. > H.M.

44. Statement-1 : The normals at the points where $t = \pm\sqrt{2}$, on the parabola $y^2 = 4ax$ subtends a right angle at the vertex of the parabola.

Statement-2 : For a normal chord of the parabola $y^2 = 4ax$ having ends t_1, t_2 we always have $t_1.t_2 = 2$.

Comprehension Type

Paragraph for Q. No. 45 to 47

If a source of light is placed at the fixed point of a parabola and if the parabola is a reflecting surface, then the ray will bounce back in a line parallel to the axis of the parabola.

45. A ray of light is coming along the line $y = 2$ from the positive direction of x-axis and strikes a concave mirror whose intersection with the x-y plane is a parabola $y^2 = 8x$, then the equation of the reflected ray is

(a) $2x + 5y = 4$ (b) $3x + 2y = 6$

(c) $4x + 3y = 8$ (d) $5x + 4y = 10$

46. A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $y^2 + 10y - 4x + 17 = 0$. After reflection, the ray must pass through the point

(a) $(-2, -5)$ (b) $(-1, -5)$ (c) $(-3, -5)$ (d) $(-4, -5)$

47. Two rays of light coming along the lines $y = 1$ and $y = -2$ form the positive direction of x-axis and strikes a concave mirror whose intersection with the x-y plane is a parabola $y^2 = x$ at A and B respectively. The reflected rays pass through a fixed point C, then the area of the triangle ABC is

(a) $\frac{21}{8}$ sq. unit (b) $\frac{19}{2}$ sq. unit

(c) $\frac{17}{2}$ sq. unit (d) $\frac{15}{2}$ sq. unit

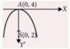
Matrix-Match Type

48. Match the following :

Column-I		Column-II	
A.	If two distinct chords of a parabola $y^2 = 4ax$ passing through the point $(a, 2a)$ are bisected by the line $x + y = 1$, then the length of the latus rectum can be	(p)	-1
B.	The parabola $y = x^2 - 5x + 4$ cuts the x-axis at P and Q. A circle is drawn through P and Q so that the origin lies outside it. The length of tangent to the circle from the origin is equal to	(q)	0
C.	If $y + b = m_1(x + a)$ and $y + b = m_2(x + a)$ are two tangents to $y^2 = 4ax$, then m_1m_2 is equal to	(r)	1
D.	If the point $(h, -1)$ is exterior to both the parabolas $y^2 = \|x\|$, then the integral part of h can be equal to	(s)	2

SOLUTIONS

1. (a) : $AS = 2 = a$.



Vertex (0, 4) lies on y-axis.

As focus is below the vertex, so it is a downward parabola of the form $x^2 = -4ay$.

or $(x - 0)^2 = -4 \times 2(y - 4) \Rightarrow x^2 + 8y = 32$

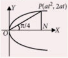
2. (b) : $(x - 2)^2 = 8(y + 1)$

Focus: $x - 2 = 0 \Rightarrow x = 2$
Similarly, $y + 1 = 2 \Rightarrow y = 1$
$\therefore$ Focus is $(2, 1)$

3. (d) : Slope of $OP = \frac{2}{t_1}$ and Slope of $OQ = \frac{2}{t_2}$

Given $OP \perp OQ \Rightarrow \frac{2}{t_1} \cdot \frac{2}{t_2} = -1 \Rightarrow t_1 t_2 = -4$

4. (a) : Since $\frac{2at}{at^2} = \tan\frac{\pi}{4}$



$\Rightarrow \quad t = 2 \quad \Rightarrow \quad P = (4a, 4a)$

$\therefore \quad OP = 4\sqrt{2}a$

5. (c): Focal distance of a point $P(x, y)$ on $y^2 = 4ax$ is $(x + a)$
i.e., $4 = x + 2 \Rightarrow x = 2$
$\therefore \quad y^2 = 8 \times 2 = 16 \Rightarrow y = \pm 4$

6. (a) : Here $a = 1$. Any tangent is $y = mx + \frac{1}{m}$.

It passes through $(-2, -1) \quad \therefore \quad 2m^2 - m - 1 = 0$

$$\Rightarrow \quad m = 1, -\frac{1}{2} \Rightarrow \tan\alpha = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} = 3$$

7. (d) : The given chord is a focal chord as it passes through focus $(a, 0)$ and we know that tangents at the extremities of a focal chord meet at right angles on the directrix.

8. (c) : Given parabola is $(x - 4)^2 = -2\left(y - \frac{9}{2}\right)$
Focus is given by $x - 4 = 0$,

$y - \frac{9}{2} = -\frac{1}{2} \Rightarrow$ Focus is $(4, 4)$.

9. (b) : $\frac{\text{Subtangent}}{x} = \frac{y}{y'x}$ and $2yy' = 4a$

$\therefore \quad y' = \frac{2a}{y} \quad \Rightarrow \quad \frac{\text{Sub tangent}}{x} = \frac{4ax}{2ax} = \frac{2}{1}$

10. (d) : Length of subnormal at point (x_1, y_1)

$= y_1 \frac{dy}{dx} = y_1\left(\frac{2a}{y_1}\right) = 2a.$

11. (c) : $x = a\left(y^2 + \frac{b}{a}y + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c$

or $\quad a\left(y + \frac{b}{2a}\right)^2 = x + \frac{b^2}{4a} - c$

$$\Rightarrow \quad \left(y + \frac{b}{2a}\right)^2 = \frac{1}{a}\left(x + \frac{b^2 - 4ac}{4a}\right)$$

$\therefore \quad$ Latus rectum $= \frac{1}{a}$

12. (c) : Required area, $\Delta = \frac{1}{2}(12 \times 3) = 18$ sq. units

13. (b) : $y = mx + k/4m$
Comparing it with the given line $y = 1 - x$,
we get $m = -1$ and $k/4m = 1 \Rightarrow k = -4$

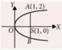
14. (a) : Any point on the parabola is $(x, x^2 + 7x + 2)$
Its distance from the line $y = 3x - 3$ is given by

$$P = \left|\frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{9 + 1}}\right| = \left|\frac{x^2 + 4x + 5}{\sqrt{10}}\right|$$

$$= \frac{x^2 + 4x + 5}{\sqrt{10}} \qquad (\because x^2 + 4x + 5 > 0 \text{ for all } x \in R)$$

Now, $\frac{dP}{dx} = 0 \Rightarrow x = -2 \quad \therefore$ Required point $\equiv (-2, -8)$

15. (b) : The parabola is $y^2 = 4x$.
Here, $a = 1$ and focus is $(1, 0)$.
The focal chord (ASB) is clearly latus rectum of parabola.
$\therefore \quad$ Length of latus rectum $= 4$



SAMURAI SUDOKU

SOLUTION - DECEMBER 2018

1	3	4	8	5	9	7	6	2				6	2	4	1	8	9	7	5	3
5	2	6	7	4	3	9	1	8				5	7	8	2	6	3	9	4	1
9	8	7	1	6	2	3	4	5				1	3	9	7	5	4	8	6	2
4	5	3	2	9	6	1	8	7				2	4	6	9	7	1	5	3	8
6	7	8	3	1	5	2	9	4				9	5	1	3	2	8	6	7	4
2	1	9	4	8	7	6	5	3				3	8	7	6	4	5	1	2	9
7	6	1	5	2	8	4	3	9	5	7	6	8	1	2	5	3	6	4	9	7
3	4	5	9	7	1	8	2	6	3	4	1	7	9	5	4	1	2	3	8	6
8	9	2	6	3	4	5	7	1	9	8	2	4	6	3	8	9	7	2	1	5
						9	5	7	1	6	4	2	3	8						
						3	4	8	2	5	9	1	7	6						
						6	1	2	7	3	8	9	5	4						
7	4	1	3	8	9	2	6	5	8	9	7	3	4	1	8	5	2	6	9	7
2	3	9	5	1	6	7	8	4	6	1	3	5	2	9	6	1	7	8	4	3
5	8	6	2	7	4	1	9	3	4	2	5	6	8	7	3	9	4	2	1	5
4	1	5	6	2	8	9	3	7				8	9	5	1	4	3	7	6	2
8	6	7	1	9	3	4	5	2				1	3	6	7	2	9	5	8	4
3	9	2	7	4	5	6	1	8				4	7	2	5	8	6	1	3	9
9	2	4	8	3	1	5	7	6				2	5	3	4	6	1	9	7	8
6	7	8	9	5	2	3	4	1				7	6	8	9	3	5	4	2	1
1	5	3	4	6	7	8	2	9				9	1	4	2	7	8	3	5	6

16. (c) : The tangent of slope m must be of the form $y = m(x + 2) + \frac{a}{m}$.

So, $2m + \frac{2}{m} = c \Rightarrow c = 2\left(m + \frac{1}{m}\right) \geq 2 \times 2$. So $c_{\min} = 4$.

17. (c) : The length of subtangent $= 2x_1 = 2 \times 4 = 8$.

18. (c) : The tangent at (x_1, y_1) is $2yy_1 - y - y_1 + x + x_1 = 0$.
Comparing it with $x + y - 1 = 0$, we get
$(x_1, y_1) \equiv (0, 1)$.

19. (a) : $x_1x_2 + y_1y_2 = a^2t_1^2t_2^2 + 4a^2t_1t_2 = a^2 - 4a^2 = -3a^2$.

20. (c) : $y^2 + 4ax\left(\frac{lx + my}{n}\right) = 0$

Now, coefficient of x^2 + coefficient of $y^2 = 0$
$\Rightarrow 4al + n = 0$.

21. (c) : Clearly, axis of parabola is the x-axis.
Corresponding value of $a = 3 - 1 = 2$.
Thus, equation of parabola is $y^2 = 8(x - 1)$.

22. (b) : Focus is $\left(\frac{7}{2}, \frac{7}{2}\right)$ and it's axis is the line $y = x$.

Corresponding value of 'a' $= \frac{1}{4}(\sqrt{1+1}) = \frac{\sqrt{2}}{4}$. Let the equation of it's directrix be $y + x + \lambda = 0$.

$$\Rightarrow \quad \frac{|3 + 4 + \lambda|}{\sqrt{2}} = 2.\frac{\sqrt{2}}{4} \quad \Rightarrow \quad \lambda = -6, -8$$

Thus equation of parabola is

$$\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{(x + y - 6)^2}{2}$$

$$\text{or} \quad \left(x - \frac{7}{2}\right)^2 + \left(x - \frac{7}{2}\right)^2 = \frac{(x + y - 8)^2}{2}$$

23. (d) : Let the mid point of chord be $P(h, k)$ then it's equation is $T = S_1$
i.e., $yk - 2a(x + h) = k^2 - 4ah$
It must pass through $(3a, a)$.
$\therefore \quad ak - 2a(3a + h) = k^2 - 4ah$
Thus, locus of P is $y^2 - 2ax - ay + 6a^2 = 0$

24. (a) : Equation of line joining A, B is
$y(t_1 + t_2) = 2(x + at_1t_2)$
It will pass through $(0, b)$, if

$$b(t_1 + t_2) = 2at_1t_2 \quad \Rightarrow \quad t_1 + t_2 - \frac{2a}{b}t_1t_2 = 0$$

25. (c) : Clearly, both the lines pass through $(-a, b)$ which is a point lying on the directrix of the parabola. Thus, $m_1m_2 = -1$.

26. (a) : If $A \equiv (at_1^2, 2at_1)$, $B \equiv ((at_2^2, 2at_2)$
$\Rightarrow \quad C \equiv (at_1t_2, a(t_1 + t_2))$
Clearly the ordinates of A, C, B are in A.P.

27. (b) : If $A \equiv (at_1^2, 2at_1)$, $B \equiv (at_2^2, 2at_2)$,
$\Rightarrow \quad C \equiv (at_1t_2, a(t_1 + t_2))$
Now, $SA = a + at_1^2$, $SB = a + at_2^2$,

$$SC = \sqrt{(at_1t_2 - a)^2 + a^2(t_1 + t_2)^2}$$
$$= a\sqrt{(t_1t_2 - 1)^2 + (t_1 + t_2)^2}$$
$$= a\sqrt{t_1^2 + t_2^2 + 1 + t_1^2t_2^2} = a\sqrt{(1 + t_1^2)(1 + t_2^2)}$$

Clearly, $SC^2 = SA \cdot SB$

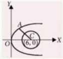
28. (c) : Let $A \equiv (2bt, bt^2)$, $B \equiv (-2bt, bt^2)$ be the extremities of the double ordinate AB. If $C(h, k)$ be it's trisection point, then $3h = 4bt - 2bt$, $3k = 2bt^2 + bt^2$.

$$\Rightarrow \quad t = \frac{3h}{2b}, t^2 = \frac{k}{b} \Rightarrow \quad \frac{k}{b} = \frac{9h^2}{4b^2}$$

Thus locus of C is $9x^2 = 4by$

29. (b) : Equation of normal in terms of m is
$y = mx - 4m - 2m^3$
If it passes through $(a, 0)$, then
$am - 4m - 2m^3 = 0 \quad \Rightarrow \quad m(a - 4 - 2m^2) = 0$
$\Rightarrow \quad m = 0, m^2 = \frac{a - 4}{2}$.
For three distinct normals, $a - 4 > 0 \quad \Rightarrow \quad a > 4$.

30. (b) : Any normal of parabola is
$y = -tx + 2t + t^3$
If it pass through $(6, 0)$, then
$-6t + 2t + t^3 = 0$
$\Rightarrow \quad t = 0, t^2 = 4 \therefore A \equiv (4, 4)$.
Thus for no common tangent,
$AC = \sqrt{4 + 16} > r \quad \Rightarrow \quad r < \sqrt{20}$

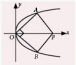


31. (a) : Let the possible point be $(t^2, 2t)$
Equation of tangent at this point is $yt = x + t^2$
It must pass through $(6, 5)$
$\therefore \quad t^2 - 5t + 6 = 0 \quad \Rightarrow \quad t = 2, 3$
Thus possible points are $(4, 4)$, $(9, 6)$.

32. (b) : Let $A \equiv (t^2, 2t)$, $B \equiv (t^2, -2t)$

$$m_{OA} = \frac{2}{t}, \quad m_{OB} = -\frac{2}{t}$$

$\Rightarrow \quad t^2 = 4$
Equation of normal AP is
$y = -2x + 4 + 8$
$\Rightarrow \quad P \equiv (6, 0)$



$$\therefore \quad \text{Ar. }(OAPB) = \frac{1}{2}(OP)(AB) = \frac{1}{2} \cdot 6 \cdot 8 = 24 \text{ sq. units.}$$

33. (a) : Equation of tangent of parabola at $(a, 2a)$ is
$y.2a = 2a(x + a)$, *i.e.*, $y - x - a = 0$.

Equation of circle touching the parabola at $(a, 2a)$ is
$(x - a)^2 + (y - 2a)^2 + \lambda\,(y - x - a) = 0$
Since, it passes through $(0, 0)$, therefore

$$a^2 + 4a^2 + \lambda\,(-a) = 0 \;\Rightarrow\; \lambda = 5a.$$

Thus required circle is $x^2 + y^2 - 7ax + ay = 0$

It's radius $= \sqrt{\frac{49}{4}a^2 + \frac{a^2}{4}} = \frac{5}{\sqrt{2}}a.$

34. (c) : Equation of parallel normal is

$$y = mx - 2m^2 - m^4$$

$\Rightarrow$ Required distance $= \frac{|c + 2m^2 + m^4|}{\sqrt{1 + m^2}}$

Here, $c = 1$
Thus required distance $= (1 + m^2)^{3/2}$

35. (d) : The line $y = x - 1$ passes through $(1, 0)$, that means it is a focal chord. Hence the required angle is $\pi/2$.

36. (d) : Normals to $y^2 = 4ax$ and $x^2 = 4by$ in terms of 'm' are $y = mx - 2am - am^3$ and $y = mx + 2b + \frac{b}{m^2}$.
For a common normal

$$2b + \frac{b}{m^2} + 2am + am^3 = 0$$

$\Rightarrow \quad am^5 + 2am^3 + 2bm^2 + b = 0$
$\therefore \quad$ There can be atmost 5 common normals.

37. (b) : Tangent to $y^2 = 4x$ in terms of m is

$$y = mx + \frac{1}{m}.$$

Normal to $x^2 = 4by$ in terms of m is

$$y = mx + 2b + \frac{b}{m^2}.$$

If these are same lines, then

$$\frac{1}{m} = 2b + \frac{b}{m^2} \quad\Rightarrow\quad 2bm^2 - m + b = 0$$

For two different tangents, we must have

$$1 - 8b^2 > 0 \quad\Rightarrow\quad |b| < \frac{1}{\sqrt{8}}$$

38. (a, b) : Equation of circle is $(x + r)^2 + y^2 = r^2$
Solving it with $y^2 + 4x = 0$, we get $x = 0$ and $x = 4 - 2r$
The circle lies completely inside the parabola, $4 - 2r$ is not less than zero $4 - 2r \geq 0 \Rightarrow r \leq 2$

39. (a, b) : Given parabola is $y^2 = 8x$ ...(i)
Here, $a = 2$. Let $P(2t^2, 4t)$ be a point on parabola
Given, $SP = 4$
$\therefore \quad a(1 + t^2) = 4 \;\Rightarrow\; 2(1 + t^2) = 4 \;\Rightarrow\; t = \pm 1$
$\therefore \quad P \equiv (2, 4)$ or $(2, -4)$
Given, $f(x + y) = f(x)f(y)$ for all x and y
Given, $f(1) = 2$ ...(ii)
From (i) and (ii), $f(2) = f(1 + 1) = f(1).f(1) = 2^2 = 4$
Similarly, $f(n) = 2^n$
Hence, $P \equiv (a_1, a_2)$ or $(a_1, -a_2)$.

40. (a, b, c) : If the parabolas have a common normal of slope 'm' (the only allowed value of m is $m = 0$) then it is given by $y = mx - 2km - km^3$
and $y = m\,(x - 1) - 2m - m^3 = mx - 3m - m^3$
$\Rightarrow \quad 2km + km^3 = 3m + m^3 \Rightarrow m = 0, m^2 = \frac{3 - 2k}{k - 1}.$
If $m^2 < 0$ then the only common normal is the axis.

$\Rightarrow \quad \frac{3 - 2k}{k - 1} < 0 \Rightarrow (k - 1)(2k - 3) > 0 \Rightarrow k > \frac{3}{2}$
or $\quad k < 1$ and $k > 0$

41. (b, c) : The parabola can be written as

$$\left(x + \frac{2}{3}\right)^2 = 2\left(y - \frac{10}{9}\right) \text{ i.e., } X^2 = 2Y\left(X = x + \frac{2}{3}, Y = y - \frac{10}{9}\right).$$

A point on axis is $\left(\frac{-2}{3}, Y\right)$ from which three normals can be drawn if $Y > 1 \Rightarrow \quad y > \frac{19}{9}$.

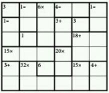
PUZZLE CORNER

MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



3	1-	6×	4-		1-
1-			3+	3	
	1			18+	
15×			20×		
3+	32×	6		15×	4+

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42. (d) : Let $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$, $R(at_3^2, 2at_3)$ be the three points on parabola.

Equation of the normal at't' is $y + xt = 2at + at^3$

The above normal is passing through (h, k)

$\Rightarrow \quad k + ht = 2at + at^3 \quad \Rightarrow \quad at^3 + (2a - h)t - k = 0$

The above is a cubic equation in 't' whose roots are $t_1, t_2, t_3 \Rightarrow \Sigma t_1 = 0$

Centroid of ΔPQR is $\left(\frac{a(t_1^2 + t_2^2 + t_3^2)}{3}, \frac{2a}{3}(t_1 + t_2 + t_3)\right)$

$$= \left(\frac{a(t_1^2 + t_2^2 + t_3^2)}{3}, 0\right)$$

Centroid lies on $y = 0$

Hence Statement - 1 is false but Statement - 2 is true.

43. (b) : Since semi latus rectum is the H.M. between focal segments

$\Rightarrow$ 'l' is H.M. of a, b

Now, G.M. >H.M.

$$\sqrt{ab} > \frac{2ab}{a+b} \Rightarrow \sqrt{ab} > l \Rightarrow ab > l^2$$

Now, A.M. > G.M.

$$\frac{a^3 + b^3}{2} > \sqrt{a^3b^3} > l^3 \Rightarrow a^3 + b^3 > 2l^3$$

44. (c) : When $P(t_1)$, $Q(t_2)$ are the ends of a normal chord then the product of slopes of AP and AQ is

$$\frac{2}{t_1} \cdot \frac{2}{t_2} = \frac{4}{t_1\left(-t_1 - \frac{2}{t_1}\right)} = \frac{-4}{t_1^2 + 2} = -1, \text{ when } t_1 = \pm\sqrt{2}$$

$\therefore$ Statement-1 is true but statement-2 is false.

45. (c) : Point of intersection of $y = 2$ and $y^2 = 8x$ is $P\left(\frac{1}{2}, 2\right)$ and focus of the parabola is $S(2, 0)$

$\therefore$ Equation of the reflected ray is $y - 0 = \frac{2-0}{1/2-2}(x-2)$

$\Rightarrow \quad y = -\frac{4}{3}(x-2) \quad \Rightarrow \quad 4x + 3y = 8$

46. (b) : $\because\ y^2 + 10y - 4x + 17 = 0$

$\Rightarrow \quad (y+5)^2 - 25 - 4x + 17 = 0$

$\Rightarrow \quad (y+5)^2 = 4x + 8 \Rightarrow (y+5)^2 = 4(x+2)$

Let $y + 5 = Y$, $x + 2 = X$, then $Y^2 = 4X$

Focus is $X = 1$, $Y = 0$, *i.e.*, $(-1, -5)$

After reflection, the ray must pass through focus $(-1, -5)$

47. (a) : Solving $y = 1$ and $y^2 = x$, we get $A \equiv (1, 1)$ and solving $y = -2$ and $y^2 = x$, we get $B \equiv (4, -2)$

$\because$ After reflection both reflected rays pass through focus of the parabola $y^2 = x$

i.e., $C \equiv \left(\frac{1}{4}, 0\right)$ $\therefore$ Required area $= \frac{21}{8}$ sq. units

48. **A** $\rightarrow$ **(r, s); B** $\rightarrow$ **(s); C** $\rightarrow$ **(p); D** $\rightarrow$ **(q)**

A. Any point is $(t, 1-t)$

The chord with this as mid point

$$y(1-t) - 2a(x+t) = (1-t)^2 - 4at$$

$\Rightarrow \quad (1-t)^2 = 2a(1-a) \geq 0 \Rightarrow 0 < a \leq 1$

$\therefore$ Length of latus Rectum $\in (0, 4]$

B. $P \equiv (1, 0)$, $Q \equiv (4, 0)$

$\therefore$ Equation of circle passes through two points is $(x-1)(x-4) + y^2 + \lambda y = 0$

$\therefore$ Length of the tangent from $(0, 0)$ is $\sqrt{4}$ *i.e.*, 2

C. $m_1m_2 = -1$

D. $1 - |h| > 0 \Rightarrow -1 < h < 1$

◆◆

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Synopsis and Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

Statistics / Probability

Statistics

INTRODUCTION

- Statistics is a branch of mathematics which deals with organization, calculation and interpretation of numerical data.
- Measures of central tendency are not sufficient to give complete information about a given data. Like measures of central tendency we want to have a single number to describe, variability. This single number is called a 'measure of dispersion'.

MEASURES OF DISPERSION

Dispersion measures the degree of scatteredness of the observations in a distribution around the central value.

The commonly used measures of dispersion are :

(i) Range
(ii) Quartile Deviation
(iii) Mean Deviation
(iv) Standard Deviation

RANGE

It is the difference between two extreme observations of the given distribution.

i.e., Range = Maximum Value – Minimum Value

MEAN DEVIATION

Mean deviation of a distribution is the arithmetic mean of the absolute deviations of the terms of the distribution from its statistical mean (A.M., median or mode).

The sum of deviations from mean ($\bar{x}$) is zero.

Mean Deviation for Ungrouped Data

Let $x_1, x_2, ..., x_n$ be the given n observations. Let $\bar{x}$ be the mean and M be the median, then

$$MD(\bar{x}) = \frac{\sum_{i=1}^{n}|x_i - \bar{x}|}{n}; \; MD(M) = \frac{\sum_{i=1}^{n}|x_i - M|}{n}$$

Mean Deviation for Grouped Data

(i) Discrete frequency distribution

$$MD(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|; \; MD(M) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M|$$

where $N = \sum_{i=1}^{n} f_i$

(ii) Continuous frequency distribution

$$MD(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|; \; MD(M) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - M|,$$

where x_i's are the mid-points of the corresponding classes.

Shortcut Method for Finding Mean Deviation

(i) **About Mean :** $MD(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|,$

where $\bar{x} = a + \frac{\sum_{i=1}^{n} f_i u_i}{N} \times h, \; u_i = \frac{x_i - a}{h}$

Here, a is the assumed mean.

(ii) About Median : $MD(M) = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - M|$,

where $M = l + \frac{\frac{N}{2} - C}{f} \times h$

Here, l, f, N and h are respectively lower limit, frequency of the median class, number of observations and class size and C denotes cumulative frequency of the class just preceding the median class.

Limitations of Mean Deviation

1. The mean deviation about median calculated for such series (where the degree of variability is very high) can not be fully relied.
2. The mean deviation about the mean is not very scientific since the sum of the deviations from the mean (if we ignored the minus signs) is more than the sum of the deviations from median.
3. The mean deviation is calculated on the basis of absolute values of the deviations and therefore, cannot be subjected to further algebraic treatment.

VARIANCE AND STANDARD DEVIATION

- Variance is the mean of the squares of all deviations of variate from their mean. It is denoted by σ^2 or var(x). Thus, if $x_1, x_2, \ldots, x_n$ be n given values of the variate and $\bar{x}$ be their mean, then

  Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$
- The positive square root of the variance of a variate X is known as its standard deviation and it is denoted by σ.

Standard Deviation for Ungrouped Data

$$S.D.(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Standard Deviation for Grouped Data

(i) Discrete frequency distribution

$$S.D.(\sigma) = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}$$

(ii) Continuous frequency distribution

$$S.D.(\sigma) = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}$$

$$\text{or } S.D.(\sigma) = \frac{1}{N}\sqrt{N\sum_{i=1}^{n} f_i x_i^2 - \left(\sum_{i=1}^{n} f_i x_i\right)^2}$$

where x_i's are mid points of class interval and $N = \sum_{i=1}^{n} f_i$.

Shortcut Method for Finding Standard Deviation

$$\sigma = \frac{h}{N}\sqrt{N\sum_{i=1}^{n} f_i y_i^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2}, \text{ where } y_i = \frac{x_i - a}{h},$$

Here, a = assumed mean, h = width of class interval and N = sum of frequency.

ANALYSIS OF FREQUENCY DISTRIBUTION

The measure of variability which is independent of units is called coefficient of variance and is denoted by C.V.

Thus, $C.V. = \frac{\sigma}{\bar{x}} \times 100$, $\bar{x} \neq 0$, where σ and $\bar{x}$ are the standard deviation and mean of the data respectively.

- For two series with equal means, the series with greater standard deviation is more variable and the series with lesser value of standard deviation is more consistent.

Note : For two groups of data taken together,

- $\text{Mean}(\bar{x}) = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$
- $\text{Variance}(\sigma^2) = \frac{1}{n_1 + n_2} \times$

  $$\left[n_1\sigma_1^2 + n_2\sigma_2^2 + \left(\frac{n_1 n_2}{n_1 + n_2}\right)(\bar{x}_1 - \bar{x}_2)^2\right],$$

  where $\bar{x}_1, \bar{x}_2$ are the respective means of the two groups having n_1 and n_2 observations.

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Probability

INTRODUCTION

Probability is a concept which numerically measures the uncertainity and therefore, of certainty of the occurrence of an event.

EXPERIMENT

An action which results in some well defined outcomes is known as an experiment.

RANDOM EXPERIMENT

An experiment is called random if it results in two or more outcomes and it is not possible to predict the outcome in advance.

OUTCOMES AND SAMPLE SPACE

- **Outcome :** A possible result of a random experiment is called its outcome.
- **Sample space :** The set of all possible outcomes of a random experiment is called sample space associated with it. It is usually denoted by S.

EVENT

A subset of the sample space associated with a random experiment is called an event.

- **Occurrence of an event :** When the outcome of an experiment satisfies the condition mentioned in the event, then we say that event has occurred.

TYPES OF EVENTS

(i) **Impossible and Sure events :** An event is said to be impossible if it contains no sample point of the sample space.

An event is said to be sure event if it is the whole sample space. The sample space S is called the sure event.

(ii) **Simple Event :** An event is said to be simple event if it consists only one sample point of sample space.

(iii) **Compound Event :** An event is said to be compound event if it consists of more than one sample point of sample space.

ALGEBRA OF EVENTS

(i) **Complementary Event :** Corresponding to every event A, we define an event "not A" which is said to occur when A does not occur. Thus, the event "not A" denoted by A' or $\bar{A}$ is called the complementary event of A.

(ii) **The Event 'A or B' :** Let A and B be two events associated with a sample space, then the event 'A or B' denoted by $A \cup B$ is the event 'either A or B or both'.

$A \cup B = \{x : x : \in A \text{ or } x \in B\}$

(iii) **The Event 'A and B' :** If A and B are two events, then the set $A \cap B$ denotes the event 'A and B'.

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

(iv) **The Event 'A but not B' :** $A - B$ is the set of all those elements which are in A but not in B. Therefore, the set $A - B$ denotes the event 'A but not B'. We know that $A - B = A \cap B'$.

MUTUALLY EXCLUSIVE EVENTS

In an experiment two or more events are said to be mutually exclusive, if happening of one rules out the happening of the other.

Events $A_1, A_2, A_3, \ldots, A_n$ of a sample space S are mutually exclusive, if $A_1 \cap A_2 \cap \ldots\ldots \cap A_n = \phi$.

EXHAUSTIVE EVENTS

The events $E_1, E_2, E_3, \ldots, E_n$ associated with a random experiment with sample space S are called exhaustive if $E_1 \cup E_2 \cup E_3 \cup\ E_n = S$.

Note :

If $E_i \cap E_j = \phi$ for $i \neq j$ *i.e.*, events E_i and E_j are pairwise disjoint and if $\bigcup_{i=1}^{n} E_i = S$, then events $E_1, E_2, \ldots, E_n$ are called mutually exclusive and exhaustive events.

Remarks :

- Number $P(\omega_i)$ associated with sample point ω_i is such that

  (i) $0 \leq P(\omega_i) \leq 1$ (ii) $\Sigma P(\omega_i) = 1\ \forall\ \omega_i \in S$

  (iii) $P(A) = \Sigma P(\omega_i)\ \forall\ \omega_i \in A$
- Let S be a sample space and E be an event, such that $n(S) = l$ and $n(E) = m$. If each outcome is equally likely, then it follows that

  $$P(E) = \frac{m}{l} = \frac{\text{Number of outcomes favourable to } E}{\text{Total possible outcomes}}$$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.
- If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$
- $P(A') = P(\text{not } A) = 1 - P(A)$

WORK IT OUT

VERY SHORT ANSWER TYPE

1. From a group of 3 boys and 2 girls, 2 children are selected at random. Describe the events at least one girl is selected.

2. In a test with maximum score 25, eleven students scored 3, 9, 5, 3, 12, 10, 17, 4, 7, 19, 21 marks respectively. Calculate the range.

3. If A and B are mutually exclusive events, then show that $P(A) \le P(\overline{B})$.

4. An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results :

	Firm A	Firm B
Number of workers	1000	1200
Average monthly wages	₹2800	₹2800
Variance of distribution of wages	100	169

In which firm, A or B is there greater variability in individual wages?

5. If A, B and C are three mutually exclusive and exhaustive events of an experiment such that $3P(A) = 2P(B) = P(C) = p$, then find $P(A)$.

SHORT ANSWER TYPE

6. If the letters of the word "ALGORITHM" are arranged at random in a row what is the probability that the letters GOR must remain together as a unit?

7. What is the probability that a non-leap year, selected at random, will have 53 tuesday or wednesday?

8. In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committe, he wins the prize. What is the probability of winning the prize in the game?

9. Let $\overline{x}$ and $\overline{y}$ are the A.M.'s of the variable x, y respectively and a, b are constants. If the variables x and y are such that $y = ax + b$, prove that $\overline{y} = a\overline{x} + b$.

10. If $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$, find $P(\overline{A}) + P(\overline{B})$.

LONG ANSWER TYPE - I

11. Find the variance and standard deviation for the following data :
65, 68, 58, 44, 48, 45, 60, 62, 60, 50.

12. Three squares of chess board are selected at random. Find the probability of getting 2 squares of one colour and other of a different colour.

13. If for a distribution $\sum(x-5) = 3$, $\sum(x-5)^2 = 43$ and the total number of items is 18, find the mean and standard deviation.

14. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then show that P (either of A or B fails) ≤ 0.5.

15. What are the odds in favour in throwing at least 8 in a single throw with two dice ?

LONG ANSWER TYPE - II

16. A panel of two judges graded seven dramatic performances by independently awarding marks as follows :

Performace	1	2	3	4	5	6	7
Marks by P	46	42	44	40	43	41	45
Marks by Q	40	38	36	35	39	37	41

Find out coefficient of variation in the marks awarded by two judges and interpret the result.

17. Two cards are drawn from a pack of 52 playing cards. What is the probability that either both are black or both are kings?

18. The mean and the standard deviation of 25 observations are 60 and 3. Later on it was decided to omit an observation which was incorrectly recorded as 50. Calculate the mean and standard deviation of the remaining 24 observations.

19. If a is a positive integer and the frequency distribution

x :	a	$2a$	$3a$	$4a$	$5a$	$6a$
f:	2	1	1	1	1	1

has a variance of 160. Determine the value of a.

20. Find the mean deviation from the mean for the following data :

Classes :	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequencies :	2	3	8	14	8	3	2

SOLUTIONS

1. Let the boys be B_1, B_2, B_3 and the girls be G_1, G_2. Let A be the event that atleast one girl is selected.

$\therefore$ $A = \{B_1G_1, B_2G_1, B_3G_1, B_1G_2, B_2G_2, B_3G_2, G_1G_2\}$

2. Here $n = 11$. The marks can be arranged in ascending order as 3, 3, 4, 5, 7, 9, 10, 12, 17, 19, 21.

Range = Maximum value – Minimum value

= 21 – 3 = 18.

3. We know that $P(A \cup B) \leq 1$

$\Rightarrow P(A) + P(B) \leq 1$

($\because$ A and B are mutually exclusive events)

$\Rightarrow P(A) \leq 1 - P(B) \Rightarrow P(A) \leq P(\overline{B})$.

4. Average monthly wages in both the firms is same *i.e.* ₹ 2800. Therefore, the firm with greater variance will have more variability. Thus, firm B has greater variability in individual wages.

5. Given $3P(A) = 2P(B) = P(C) = p$

$\Rightarrow P(A) = \frac{p}{3}, P(B) = \frac{p}{2}$ and $P(C) = p$

Also, $P(A) + P(B) + P(C) = 1$

[$\because$ Since A, B and C are mutually exclusive and exhaustive events.]

$\Rightarrow \frac{p}{3} + \frac{p}{2} + p = 1 \Rightarrow \frac{11}{6}p = 1 \Rightarrow p = \frac{6}{11}$

Hence, $P(A) = \frac{p}{3} = \frac{1}{3} \times \frac{6}{11} = \frac{2}{11}$.

6. There are 9 letters in the word "ALGORITHM". These 9 letters can be arranged in a row in 9! ways.

$\therefore$ Total number of elementary events = 9!

Considering GOR as one letter, there are 7 letters which can be arranged in a row in 7! ways.

$\therefore$ Favourable number of elementary events = 7!

Hence, required probability $= \frac{7!}{9!} = \frac{1}{72}$.

7. A non-leap year has 365 years, 364 days make 52 weeks and 1 day is left. In each week, there is one tuesday and one wednesday.

So, we have to find the probability that the remaining day is tuesday or wednesday.

Out of 7 days, the number of outcomes favourable to the event 'Tuesday or Wednesday' is 2.

$\therefore$ Required probability $= \frac{2}{7}$.

8. Six numbers can be chosen from 20 numbers in ${}^{20}C_6$ ways.

$\therefore$ Total number of elementary events $= {}^{20}C_6 = 38760$

Now, a person wins the prize if six selected numbers match with the six numbers already fixed by the committe.

$\therefore$ Favourable number of ways = 1

Hence, required probability $= \frac{1}{38760}$.

9. Let number of observations for variable $x = n$

Since, $y = ax + b$

For each value of x, y will have one value.

$\therefore$ Number of observations for variable $y = n$

Now $\bar{y} = \frac{\sum y}{n} = \frac{\sum(ax+b)}{n}$ [$\because y = ax + b$]

$= \frac{\sum ax + \sum b}{n} = \frac{a\sum x + bn}{n} = \frac{a\sum x}{n} + b = a\bar{x} + b$

10. We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.6 = P(A) + P(B) - 0.2$

[$\because P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$]

$\Rightarrow P(A) + P(B) = 0.8$

Now, $P(\overline{A}) + P(\overline{B}) = (1 - P(A)) + (1 - P(B))$

$= 2 - (P(A) + P(B)) = 2 - 0.8 = 1.2$

11. Let $\bar{x}$ be the mean of the given set of observations. Then,

$$\bar{x} = \frac{65+68+58+44+48+45+60+62+60+50}{10} = \frac{560}{10} = 56$$

Computation of variance

x_i	$x_i - \bar{x} = x_i - 56$	$(x_i - \bar{x})^2$
65	9	81
68	12	144
58	2	4
44	−12	144
48	−8	64
45	−11	121
60	4	16
62	6	36
60	4	16
50	−6	36
		$\sum(x_i - \bar{x})^2 = 662$

Clearly, $n = 10$ and $\sum(x_i - \bar{x})^2 = 662$

$\therefore$ Variance $= \frac{1}{n}\sum(x_i - \bar{x})^2 = \frac{662}{10} = 66.2$

$\therefore$ Standard deviation $(\sigma) = \sqrt{\text{Variance}} = \sqrt{66.2} = 8.13$

12. In a Chess board, there are 64 squares of which 32 are white and 32 are black.

Out of 64 squares 3 squares can be chosen in ${}^{64}C_3$ ways.

Since 2 of one colour and 1 ways of other colour can be 2W, 1B or 1W, 2B.

Therefore, the number of ways of selecting 2 squares of one colour and one other colour is

${}^{32}C_2 \times {}^{32}C_1 + {}^{32}C_1 \times {}^{32}C_2 = 2({}^{32}C_2 \times {}^{32}C_1)$

$\therefore$ Favourable number of ways $= 2({}^{32}C_2 \times {}^{32}C_1)$

Hence, required probability $= \frac{2({}^{32}C_2 \times {}^{32}C_1)}{{}^{64}C_3} = \frac{16}{21}$.

13. Here, total number of observations is 18 *i.e.*, $n = 18$

Given, $\sum(x - 5) = 3 \Rightarrow \sum x - \sum 5 = 3$

$\Rightarrow \sum x - 5 \times 18 = 3 \Rightarrow \sum x = 93$

$\therefore$ Mean $= \bar{x} = \dfrac{\sum x}{n} = \dfrac{93}{18}$

Also, $\sum(x - 5)^2 = 43$ (Given) $\Rightarrow \sum(x^2 - 10x + 25) = 43$

$\Rightarrow \sum x^2 - 10\sum x + \sum 25 = 43$

$\Rightarrow \sum x^2 - 10 \times 93 + 25 \times 18 = 43$

$\Rightarrow \sum x^2 = 43 + 930 - 450 \Rightarrow \sum x^2 = 523$

$\therefore$ Variance $(\sigma^2) = \dfrac{\sum x^2}{n} - (\bar{x})^2 = \dfrac{523}{18} - \left(\dfrac{93}{18}\right)^2$

$= \dfrac{523 \times 18 - (93)^2}{(18)^2} = \dfrac{765}{324} = 2.36$

$\therefore$ Standard deviation $= \sqrt{2.36} = 1.54$ (approx).

14. Let E and F be the events that A and B passes in an examination.

We are given that $P(\bar{E}) = 0.2$ and $P(\bar{F}) = 0.3$

$\Rightarrow 1 - P(E) = 0.2$ and $1 - P(F) = 0.3$

$\Rightarrow P(E) = 0.8$ and $P(F) = 0.7$

Also, $P(E \cup F) \leq 1$

$\Rightarrow P(E) + P(F) - P(E \cap F) \leq 1$

$\Rightarrow 0.8 + 0.7 - P(E \cap F) \leq 1$

$\Rightarrow -P(E \cap F) \leq 1 - 0.8 - 0.7 \Rightarrow -P(E \cap F) \leq -0.5$

$\Rightarrow 1 - P(E \cap F) \leq 0.5 \Rightarrow P(\overline{E \cap F}) \leq 0.5$

$\Rightarrow P(\bar{E} \cup \bar{F}) \leq 0.5$

$\Rightarrow P$(either of A or B fails) ≤ 0.5

15. Let S be the sample space and E be the event of occurrence of a total of at least 8.

$S = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$

$\therefore \ n(S) = 6 \times 6 = 36$

Throwing at least 8 with two dice means throwing 8 or 9 or 10 or 11 or 12.

$\therefore \ E = \{(6, 2), (5, 3), (4, 4), (3, 5), (2, 6), (6, 3), (5, 4), (4, 5), (3, 6), (6, 4), (5, 5), (4, 6), (6, 5), (5, 6), (6, 6)\}$

$\therefore \ n(E) = 15$

Now, $n(E') = 36 - 15 = 21$

$\therefore$ Odds in favour of $E = \dfrac{n(E)}{n(E')} = \dfrac{15}{21} = \dfrac{5}{7} = 5:7.$

16. For the judge P, mean $\bar{x} = \dfrac{\sum x_i}{7}$

$= \dfrac{46 + 42 + 44 + 40 + 43 + 41 + 45}{7} = \dfrac{301}{7} = 43$

For the judge Q, mean $\bar{y} = \dfrac{\sum y_i}{7}$

$= \dfrac{40 + 38 + 36 + 35 + 39 + 37 + 41}{7} = \dfrac{266}{7} = 38$

Marks by $P(x_i)$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	Marks by $Q\ (y_i)$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
40	–3	9	35	–3	9
41	–2	4	36	–2	4
42	–1	1	37	–1	1
43	0	0	38	0	0
44	1	1	39	1	1
45	2	4	40	2	4
46	3	9	41	3	9
Total	0	28		0	28

For judge P, $\sigma_x = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n}} = \sqrt{\dfrac{28}{7}} = 2$

$C.V._x = \dfrac{\sigma_x}{\bar{x}} \times 100 = \dfrac{2}{43} \times 100 = 4.65$ (approx.)

For judge Q, $\sigma_y = \sqrt{\dfrac{\Sigma(y_i - \bar{y})^2}{n}} = \sqrt{\dfrac{28}{7}} = 2$

$C.V._y = \dfrac{\sigma_y}{\bar{y}} \times 100 = \dfrac{2}{38} \times 100 = 5.26$ (approx.)

Hence, judge P is more liberal in giving marks (as mean is higher) and is more consistent (as *C.V.* is lower).

17. Let S be the sample space, A be the event that the two cards drawn are black, and B be the event that the two cards drawn are kings. Then, $A \cap B$ denotes the event that the two cards drawn are black and kings.

Now, $n(S)$ = number of ways of drawing two cards out of 52 cards = ${}^{52}C_2$

$n(A)$ = number of ways in which 2 black cards can be drawn from 26 black cards = ${}^{26}C_2$

$n(B)$ = number of ways in which 2 kings can be drawn from 4 kings = ${}^{4}C_2$

$n(A \cap B)$ = number of ways of drawing 2 cards that are both black and kings = ${}^{2}C_2$

$\therefore \quad P(A) = \dfrac{{}^{26}C_2}{{}^{52}C_2}, P(B) = \dfrac{{}^{4}C_2}{{}^{52}C_2}$ and $P(A \cap B) = \dfrac{{}^{2}C_2}{{}^{52}C_2}$

Required probability = P(both are black or both are kings)

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

[By addition theorem]

$= \dfrac{{}^{26}C_2}{{}^{52}C_2} + \dfrac{{}^{4}C_2}{{}^{52}C_2} - \dfrac{{}^{2}C_2}{{}^{52}C_2} = \dfrac{650}{2652} + \dfrac{12}{2652} - \dfrac{2}{2652} = \dfrac{55}{221}$

18. Given $n = 25$, mean = 60 and standard deviation = 3 *i.e.*, variance = 9.

Now mean = 60 $\Rightarrow \dfrac{\sum_{i=1}^{25} x_i}{25} = 60 \Rightarrow \sum_{i=1}^{25} x_i = 1500$

Also, variance = 9 $\Rightarrow \dfrac{\sum_{i=1}^{25} x_i^2}{25} - (60)^2 = 9$

$\Rightarrow \sum_{i=1}^{25} x_i^2 = 25(9+3600) = 25 \times 3609 = 90225$

When the observation 50 is omitted, then 24 observations are left.

Correct $\sum_{i=1}^{24} x_i = 1500 - 50 = 1450$

$\therefore$ Correct mean $= \bar{x} = \dfrac{\sum_{i=1}^{24} x_i}{24} = \dfrac{1450}{24} = 60.42$ (approx.)

Correct $\sum_{i=1}^{24} x_i^2 = 90225 - (50)^2 = 87725$

$\therefore$ Correct $\sigma^2 = \dfrac{\sum_{i=1}^{24} x_i^2}{24} - \bar{x}^2 = \dfrac{87725}{24} - \left(\dfrac{1450}{24}\right)^2$

$= 3655.208 - 3650.174 = 5.034$

$\Rightarrow$ Correct $\sigma = \sqrt{5.0348} = 2.24$

19. Computation of Variance :

x_i	f_i	$f_i x_i$	$f_i x_i^2$
a	2	$2a$	$2a^2$
$2a$	1	$2a$	$4a^2$
$3a$	1	$3a$	$9a^2$
$4a$	1	$4a$	$16a^2$
$5a$	1	$5a$	$25a^2$
$6a$	1	$6a$	$36a^2$
	$N = \Sigma f_i = 7$	$\Sigma f_i x_i = 22a$	$\Sigma f_i x_i^2 = 92a^2$

Here, $N = 7$, $\Sigma f_i x_i = 22a$, $\Sigma f_i x_i^2 = 92a^2$

Now, Variance = 160 [Given]

$\Rightarrow 160 = \left(\dfrac{1}{N}\Sigma f_i x_i^2\right) - \left(\dfrac{1}{N}\Sigma f_i x_i\right)^2$

$\Rightarrow 160 = \dfrac{92a^2}{7} - \left(\dfrac{22a}{7}\right)^2 \Rightarrow 160 = \dfrac{644a^2 - 484a^2}{49}$

$\Rightarrow 160 = \dfrac{160a^2}{49} \Rightarrow a^2 = 49 \Rightarrow a = 7.$

20. Computation of Mean Deviation from Mean

Classes	Mid-values x_i	Frequencies f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$ $= \lvert x_i - 45\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
10-20	15	2	30	30	60
20-30	25	3	75	20	60
30-40	35	8	280	10	80
40-50	45	14	630	0	0
50-60	55	8	440	10	80
60-70	65	3	195	20	60
70-80	75	2	150	30	60
		$N = \Sigma f_i = 40$	$\Sigma f_i x_i = 1800$		$\Sigma f_i \lvert x_i - \bar{x}\rvert = 400$

Clearly $N = 40$ and $\Sigma f_i x_i = 1800$

$\therefore \bar{x} = \dfrac{\Sigma f_i x_i}{N} = \dfrac{1800}{40} = 45$

From the above table, we get

$\Sigma f_i \lvert x_i - \bar{x}\rvert = 400$ and $N = \Sigma f_i = 40$

$\therefore MD(\bar{x}) = \dfrac{1}{N}\Sigma f_i \lvert x_i - \bar{x}\rvert = \dfrac{400}{40} = 10$

◆

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MPP-9 MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Conic Sections

Total Marks : 80

Time Taken : 60 Min.

Only One Option Correct Type

1. The equation of a circle C_1 is $x^2+y^2-4x-2y-11=0$. A circle C_2 of radius 1 rolls on the outside of the circle C_1. The locus of the centre of circle C_2 has the equation

(a) $x^2+y^2-4x-2y-20=0$
(b) $x^2+y^2+4x+2y-20=0$
(c) $x^2+y^2-3x-y-11=0$
(d) none of these

2. Let d_1 and d_2 be the lengths of perpendiculars drawn from foci S and S' of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent at any point P on the ellipse. Then, $SP : S'P =$

(a) $d_1 : d_2$ (b) $d_2 : d_1$
(c) $d_1^2 : d_2^2$ (d) none of these

3. If radii of director circles of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b_1^2}=1$ are $2r$ and r respectively and e_e and e_h be the eccentricities of the ellipse and hyperbola respectively, then

(a) $2e_h^2-e_e^2=6$ (b) $e_e^2-4e_h^2=6$
(c) $4e_h^2-e_e^2=6$ (d) none of these

4. If $\left(m_r, \frac{1}{m_r}\right)$, $r = 1, 2, 3, 4$ are concyclic points and $f(x)=\begin{cases} x, \text{ when } x \text{ is rational} \\ 1-x, \text{ when } x \text{ is irrational} \end{cases}$ and a is a point where $f(x)$ is continuous, then $m_1m_2m_3m_4 =$

(a) a (b) $2a$
(c) $-2a$ (d) none of these

5. The point $(a, 2a)$ is an interior point of the region bounded by the parabola $y^2 = 16x$ and the double ordinate through the focus. Then, a belongs to the open interval

(a) $a < 4$ (b) $0 < a < 4$
(c) $0 < a < 2$ (d) $a > 4$

6. Normals drawn to $y^2 = 4ax$ at the points where it is intersected by the line $y = mx + c$ intersect at P. The foot of the another normal drawn to the parabola from the point P is

(a) $(a/m^2, -2a/m)$ (b) $(9a/m, -6a/m)$
(c) $(am^2, -2am)$ (d) $(4a/m^2, -4a/m)$

One or More Than One Option(s) Correct Type

7. The focal chord to $y^2 = 64x$ is tangent to $(x-4)^2 + (y-2)^2 = 4$, then the possible values of the slope of this chord is/are

(a) 0 (b) $-12/35$ (c) $12/35$ (d) $35/12$

8. If length of latus rectum of ellipse $E_1 : 2(x+y-1)^2 + (x-y+3)^2 = 4$ and $E_2 : \frac{x^2}{\lambda}+\frac{y^2}{\lambda^2}=1$ $(0<\lambda<1)$ are equal, then

(a) area of ellipse E_2 is $\frac{\pi}{\sqrt{2}}$ sq. units
(b) area of director circle of E_2 is $\pi\left(2^{-\frac{1}{3}}+2^{-\frac{2}{3}}\right)$ sq. units
(c) area of E_1 is 2π sq. units
(d) none of these

9. Equation $x^2-2x-2y+5=0$ represents

(a) a circle with centre (1, 1)
(b) a parabola with vertex (1, 2)
(c) a parabola with directrix $y=\frac{5}{2}$
(d) a parabola with directrix $y=-\frac{1}{3}$

10. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ at four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, and $S(x_4, y_4)$, then
(a) $x_1 + x_2 + x_3 + x_4 = 0$ (b) $y_1 + y_2 + y_3 + y_4 = 0$
(c) $x_1x_2x_3x_4 = c^4$ (d) $y_1y_2y_3y_4 = c^4$

11. The distance of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ from the centre is 2. Then the eccentric angle of the point is
(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$

12. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts each of the circles $x^2 + y^2 - 4 = 0$, $x^2 + y^2 - 6x - 8y + 10 = 0$ and $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter, then
(a) $c = -4$ (b) $g + f = c - 1$
(c) $g^2 + f^2 - c = 17$ (d) $gf = 6$

13. If $x, y \in R$, then the equation $3x^4 - 2(19y + 8)x^2 + 361y^2 + 2(100 + y^4) + 64 = 2(190y + 2y^2)$ represents in rectangular cartesian system a/an
(a) parabola (b) hyperbola
(c) circle (d) ellipse

Comprehension Type

A parabola is drawn through two given points $A(1, 0)$ and $B(-1, 0)$ such that its directrix always touches the circle $x^2 + y^2 = 4$. Then,

14. The locus of focus of the parabola is
(a) $\frac{x^2}{4}+\frac{y^2}{3}=1$ (b) $\frac{x^2}{4}+\frac{y^2}{5}=1$
(c) $\frac{x^2}{3}+\frac{y^2}{4}=1$ (d) $\frac{x^2}{5}+\frac{y^2}{4}=1$

15. The maximum possible length of semilatus rectum is
(a) $2+\sqrt{3}$ (b) $3+\sqrt{3}$ (c) $4+\sqrt{3}$ (d) $1+\sqrt{3}$

Matrix Match Type

16. Let the foci of the hyperbola $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$ be the vertices of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the foci of the ellipse be the vertices of the hyperbola. Let the eccentricities of the ellipse and hyperbola be e_E and e_H respectively. Then match the following columns:

	Column-I		Column-II
P.	b/B is equal to	1.	1
Q.	$e_H + e_E$ is always greater than	2.	2
R.	If the angle between the asymptotes of the hyperbola is $2\pi/3$, then $4e_E$ is equal to	3.	3
S.	If $e_E = 1/2$ and (x, y) is the point of intersection of the ellipse and the hyperbola, then $9x^2/2y^2$ is	4.	4

	P	Q	R	S
(a)	2	2,3	1	2
(b)	1	1,2	2	4
(c)	3	1,4	2	4
(d)	4	2,4	3	1

Numerical Answer Type

17. Tangent are drawn from any point on the hyperbola $x^2/9 - y^2/4 = 1$ to the circle $x^2 + y^2 = 9$. If the locus of the mid-point of the chord of contact is $a(x^2 + y^2)^2 = bx^2 - cy^2$, then the value of $a^2 + b^2 + c^2$ is

18. A circle $(x - \alpha)^2 + (y - \beta)^2 = 8$ is inscribed in a parabola $y^2 = 4x$, then the value of $\alpha + \beta$ is

19. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\pi/3$, then the eccentricity of conjugate hyperbola is

20. If (5, 12) and (24, 7) are the foci of an ellipse passing through the origin and if e is the eccentricity, then the value of $\left[\frac{1}{e}\right]$, where [.] represents the greatest integer function, is

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SELF CHECK

No. of questions attempted ……
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74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CONCEPT BOOSTERS

Indefinite Integrals

Class XII

This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

*ALOK KUMAR

Let $F(x)$ be a differentiable function of x such that $\frac{d}{dx}[F(x)] = f(x)$. Then $F(x)$ is called integral of $f(x)$.

Symbolically, it is written as $\int f(x)dx = F(x)$.

Here, $F(x)$ is also called anti-derivative (or primitive function) of $f(x)$.

As the differential coefficient of a constant is zero, we have

$$\frac{d}{dx}(F(x)) = f(x) \Rightarrow \frac{d}{dx}[F(x)+c] = f(x)$$

$\therefore \int f(x)dx = F(x) + c$, where c is called the constant of integration and can take any real value.

SOME FUNDAMENTAL INTEGRALS

- $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$
- $\int \frac{1}{x} dx = \log|x| + c, x \neq 0$
- $\int e^x\, dx = e^x + c$
- $\int a^x\, dx = \frac{a^x}{\log a} + c\ (a > 0)$
- $\int \sin x\, dx = -\cos x + c$
- $\int \cos x\, dx = \sin x + c$
- $\int \sec^2 x\, dx = \tan x + c$
- $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$
- $\int \operatorname{cosec} x \cdot \cot x\, dx = -\operatorname{cosec} x + c$
- $\int \sec x \cdot \tan x\, dx = \sec x + c$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
- $\int \cot x\, dx = \log|\sin x| + c = -\log|\operatorname{cosec} x| + c$
- $\int \tan x\, dx = -\log|\cos x| + c = \log|\sec x| + c$
- $\int \sec x\, dx = \log|\sec x + \tan x| + c$
- $\int \operatorname{cosec} x\, dx = \log|(\cot x - \operatorname{cosec} x)| + c$
  $= \log\left|\tan\frac{x}{2}\right| + c$

STANDARD FORMULAE

- $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

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*Alok Kumar, a B Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

- $\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c$
- $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$
- $\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

METHODS OF INTEGRATION

There are three methods for finding integral of a function.

(i) Integration by substitution

If integral is of the form $\int f(g(x))g'(x)dx$, then put $g(x) = t$, provided $\int f(t)dt$ exists.

Integrals	Substitution
$\int\frac{f'(x)}{f(x)}dx$	$f(x) = t$
$\int\frac{f'(x)}{\sqrt{f(x)}}dx$	$f(x) = t$

Standard Substitutions

For terms of the form

- $x^2 + a^2$ or $\sqrt{x^2+a^2}$, put $x = a\tan\theta$ or $a\cot\theta$
- $x^2 - a^2$ or $\sqrt{x^2-a^2}$, put $x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$
- $a^2 - x^2$ or $\sqrt{a^2-x^2}$, put $x = a\sin\theta$ or $a\cos\theta$
- If both $\sqrt{a+x}, \sqrt{a-x}$ are present, put $x = a\cos\theta$.
- $\sqrt{(x-a)(b-x)}$, put $x = a\cos^2\theta + b\sin^2\theta$
- $\left(\sqrt{x^2+a^2}\pm x\right)^n$ or $\left(x\pm\sqrt{x^2-a^2}\right)^n$, put the expression within the bracket = t.
- $(x+a)^{-1-\frac{1}{n}}(x+b)^{-1+\frac{1}{n}}$ or $\left(\frac{x+b}{x+a}\right)^{\frac{1}{n}-1}\frac{1}{(x+a)^2}$ $(n \in N, n > 1)$, put $\frac{x+b}{x+a}=t$.
- $\frac{1}{(x+a)^{n_1}(x+b)^{n_2}}$, $n_1, n_2 \in N$ (and > 1), put $(x + a) = t(x + b)$

(ii) Integration by parts

If u and v be two functions of x, then integral product of these two functions is given by

$$\int uv\,dx = u\int v\,dx-\int\left(\frac{du}{dx}\int v\,dx\right)dx$$

Usually we use the following preference order for the first function

(Inverse, Logarithmic, Algebraic, Trigonometric, Exponent)

In the above stated order, the function on the left is always chosen as the first function. This rule is called as ILATE.

(iii) Integration by partial fractions

A function of the form $p(x)/q(x)$, where $p(x)$ and $q(x)$ are polynomials, is called a rational function. Depending on the nature of $p(x)$ and $q(x)$ integration can be done by the following process:

1. If degree $(p(x))$ < degree $(q(x))$ *i.e.*, $f(x)=\frac{mx+n}{(x-a)(x-b)}$, $a \neq b$ then we write $\frac{mx+n}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$, A and B being constants.
2. If degree $(p(x))$ = degree $(q(x))$ or degree $(p(x))$ > degree $(q(x))$ of non-repeated linear factors, *i.e.*, $f(x)=\frac{mx^2+nx+l}{(x-a)(x-b)}$, $a \neq b$ then we write $\frac{mx^2+nx+l}{(x-a)(x-b)}=1+\frac{A}{x-a}+\frac{B}{x-b}$
3. If denominator $q(x)$ contains linear factors, some of which are repeated, *i.e.*, integrand is of the form $\frac{p(x)}{(x-a)(x-b)^2}$, then we write the integrand as $\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{(x-b)^2}$.

Algebraic Integrals

- In integrals of the form

  $$\int\frac{px+q}{ax^2+bx+c}dx, \int\frac{px+q}{\sqrt{ax^2+bx+c}}dx,$$

  $$\int(px+q)\sqrt{ax^2+bx+c}\,dx$$

  Take $px + q = l$ (diff. coefficient of $ax^2 + bx + c$) + m

  Find l and m by comparing the coefficient of x and constant term on both sides of the identity.

- In integrals of the type

  $$\frac{ax^2+bx+c}{px^2+qx+r} \text{ or } \frac{ax^2+bx+c}{\sqrt{px^2+qx+r}}$$

  Take $ax^2 + bx + c = M(px^2 + qx + r) + N(2px + q) + R$

  Find M, N, and R. The integration reduces to integration of three independent functions.

Integration of Irrational Algebraic Fractions

- Irrational functions of $(ax + b)^{1/n}$ and x can be easily evaluated by the substitution $t^n = ax + b$.

  $$\therefore\quad \int f(x,(ax+b)^{1/n})dx=\int f\left(\frac{t^n-b}{a},t\right)\frac{nt^{n-1}}{a}dt$$

- In integral of the type $\int \frac{dx}{(x-k)^r \sqrt{ax^2+bx+c}}$, take $x-k=\frac{1}{t}$.

  This substitution will reduce the given integral to $-\int \frac{t^{r-1}dt}{\sqrt{At^2+Bt+C}}$

- To integrate $\frac{1}{(Ax^2+B)\sqrt{Cx^2+D}}$, we first put $x=\frac{1}{t}$,

  $$\therefore \quad \int \frac{dx}{(Ax^2+B)\sqrt{Cx^2+D}} = -\int \frac{tdt}{(A+Bt^2)\sqrt{C+Dt^2}}$$

  Now, put $C+Dt^2=u^2$, to get $\int \frac{du}{u^2 \pm a^2}$.

- In integral of the type $\int \frac{(ax^2+bx+c)dx}{(dx+e)\sqrt{fx^2+gx+h}}$

  Consider $ax^2+bx+c = A_1(dx+e)(2fx+g) + B_1(dx+e)+C_1$,

  Find A_1, B_1 and C_1 and given integral will reduce to the form

  $$A_1\int \frac{(2fx+g)}{\sqrt{fx^2+gx+h}}dx + B_1\int \frac{dx}{\sqrt{fx^2+gx+h}} + C_1\int \frac{dx}{(dx+e)\sqrt{fx^2+gx+h}}.$$

TRIGONOMETRIC INTEGRALS

- Integrals of the form $\int R(\sin x, \cos x)dx$

  Here R is a rational function of $\sin x$ and $\cos x$ then this can be translated into integrals of a rational function by the substitution $\tan\left(\frac{x}{2}\right)=t$.

  In this case

  $$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2},$$

  $$x = 2\tan^{-1} t \ ; \quad dx = \frac{2\,dt}{1+t^2}$$

  Sometimes, instead of the substitution $\tan\frac{x}{2}=t$, it is more advantageous to make the substitution $\cot\frac{x}{2}=t$.

- **Integrals of the form:**

  (i) $\int \frac{p\cos x + q\sin x + r}{a\cos x + b\sin x + c}dx$

  (ii) $\int \frac{p\cos x + q\sin x}{a\cos x + b\sin x}dx$

  Rule for (i) : In this integral express numerator as l(denominator) + m(derivative of denominator) + n. Find l, m, n and split the integral into sum of three integrals *i.e.*,

  $$l\int dx + m\int \frac{\text{derivative of denominator}}{\text{denominator}}dx + n\int \frac{dx}{a\cos x + b\sin x + c}$$

  Rule for (ii) : Express numerator as l (denominator) + m(derivative of denominator) and find l and m.

- **Integration of the Type:** $\int (\sin^m x \cos^n x)dx$, where m and $n \in$ natural numbers.

(i) If one of them is odd, then substitute for term of even power.

(ii) If both are odd, substitute either of the term.

(iii) If both are even, use trigonometric identities only.

PROBLEMS

Single Correct Answer Type

1. If $I=\int \frac{dx}{e^x+4e^{-x}} = f(x)+c$, then $f(x)$ equals

(a) $2\tan^{-1}(2e^x)$ (b) $\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)$

(c) $2\tan^{-1}\frac{e^x}{2}$ (d) $\frac{1}{2}\tan^{-1}(2e^{2x})$

2. $I=\int \frac{\sin x}{\sin x - \cos x}dx$ is equal to

(a) $\frac{x}{2}-\frac{1}{2}\log(\sin x - \cos x)+c$

(b) $\frac{x}{2}+\frac{1}{2}\log(\sin x - \cos x)+c$

(c) $\frac{x}{2}+\frac{1}{2}\log(\sin x + \cos x)+c$

(d) none of these

3. $I=\int \frac{\cos 5x + \cos 4x}{1-2\cos 3x}dx$ is equal to

(a) $\frac{\sin 2x}{2}-\sin x+c$ (b) $-\frac{\sin 2x}{2}+\sin x+c$

(c) $-\frac{\sin 2x}{2}-\sin x+c$ (d) $\frac{\sin 2x}{2}+\sin x+c$

4. $I=\int \tan^3 2x \sec 2x dx =$

(a) $\sec^3 2x + 3\sec 2x + c$ (b) $\frac{1}{6}(\sec^3 2x - 3\sec 2x)+c$

(c) $(\sec^3 2x - 3\sec 2x)+c$ (d) none of these

5. $I=\int \cos^3 x e^{\log(\sin x)} dx$ is equal to

(a) $-\frac{\sin^4 x}{4}+c$ (b) $-\frac{\cos^4 x}{4}+c$

(c) $\frac{e^{\sin x}}{4}+c$ (d) none of these

6. $I=\int \frac{\log(x+1)-\log x}{x(x+1)} dx$ equals

(a) $-\log\left(\frac{x+1}{x}\right)+c$ (b) $-\log\left[\log\left(\frac{x+1}{x}\right)\right]+c$

(c) $-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2+c$ (d) none of these

7. If $\int \frac{1}{x\sqrt{1-x^3}} dx = a\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|+b$, then a is equal to

(a) 1/3 (b) 2/3 (c) –1/3 (d) –2/3

8. $I=\int \frac{\cos 2x}{\cos x} dx$ is equal to

(a) $2 \sin x + \log(\sec x - \tan x) + c$

(b) $2 \sin x - \log(\sec x - \tan x) + c$

(c) $2 \sin x + \log(\sec x + \tan x) + c$

(d) $2 \sin x - \log(\sec x + \tan x) + c$

9. $\int x^2 \sin x \, dx$ is equal to

(a) $x^2 \sin x - 2x \cos x + c$ (b) $x^2 \sin x + c$

(c) $-x^2 \cos x + 2x \sin x + 2 \cos x + c$

(d) $-x^2 \sin x - 2x \cos x + \sin x + c$

10. $I=\int (1-x)^{23} x \, dx =$

(a) $\frac{x^{23}}{23}+\frac{x^{24}}{24}+c$ (b) $\frac{(x-1)^{23}}{23}+\frac{(x-1)^{24}}{24}+c$

(c) $\frac{(1-x)^{25}}{25}-\frac{(1-x)^{24}}{24}+c$ (d) none of these

11. $I=\int \frac{dx}{\cos^6 x+\sin^6 x}$ is equal to

(a) $\log_e(\tan x - \cot x) + c$ (b) $\log_e(\cot x + \tan x) + c$

(c) $\tan^{-1}(\tan x - \cot x) + c$ (d) $\tan^{-1}(2 \cot 2x) + c$

12. $I=\int \frac{(x+x^{2/3}+x^{1/6})}{x(1+x^{1/3})} dx$ is equal to

(a) $\frac{3}{2}x^{2/3}+6\tan^{-1}(x^{1/6})+c$

(b) $\frac{3}{2}x^{2/3}-6\tan^{-1}(x^{1/6})+c$

(c) $\frac{3}{2}x^{2/3}+\tan^{-1}(x^{1/6})+c$ (d) none of these

13. $I=\int (e^{\log x}+\sin x)\cos x \, dx$ is equal to

(a) $x \sin x + \cos x - \sin^2 x + c$

(b) $x \cos x - \sin^2 x + c$

(c) $x \sin x + \cos x - (\cos^2 x)/2 + c$

(d) $x^2 \sin x + \cos x - \sin^3 x + c$

14. $\int \frac{1}{4\cos^3 x-3\cos x} dx$ is equal to

(a) $\frac{1}{3}\ln|\sec 3x-\tan 3x|+c$

(b) $\frac{1}{3}\ln|\sec 3x+\tan 3x|+c$

(c) $\frac{1}{4}\ln|\sec 3x+\tan 3x|+c$

(d) $\frac{1}{4}\ln|\sec 3x-\tan 3x|+c$

15. $\int \frac{1}{x[(\ln x)^2+4\ln x-1]} dx$ is equal to

(a) $\frac{1}{\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

(b) $\frac{1}{2}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

(c) $\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x-2+\sqrt{5}}\right|+c$

(d) $\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c$

16. $I=\int \frac{\sin x+\cos x}{9+16\sin 2x} dx$ is equal to

(a) $\frac{1}{40}\ln\left|\frac{5+4(\sin x-\cos x)}{5-4(\sin x-\cos x)}\right|+c$

(b) $\frac{1}{40}\ln\left|\frac{5-4(\sin x-\cos x)}{5+4(\sin x-\cos x)}\right|+c$

(c) $\frac{1}{40}\ln\left|\frac{5+4(\sin x+\cos x)}{5-4(\sin x+\cos x)}\right|+c$

(d) $\frac{1}{40}\ln\left|\frac{5-4(\sin x+\cos x)}{5+4(\sin x-\cos x)}\right|+c$

17. For what value of a and b, the equation $\int \frac{dx}{1+\sin x} = \tan\left(\frac{x}{2}+a\right)+b$ holds good?

(a) $a=-\frac{5\pi}{4}$, b is any arbitrary constant

(b) $a=\frac{5\pi}{4}$, b is any arbitrary constant

(c) $a=-\frac{\pi}{4}$, b is any arbitrary constant

(d) $a=\frac{\pi}{4}$, b is any arbitrary constant

18. $I=\int\frac{\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{1+x^2}}dx$ is equal to

(a) $\frac{1}{2}\left[\ln\left(x-\sqrt{x^2+1}\right)\right]^2+c$

(b) $\left[\ln\left(x-\sqrt{x^2+1}\right)\right]^2+c$

(c) $\frac{1}{2}\left[\ln\left(x+\sqrt{x^2+1}\right)\right]^2+c$

(d) $\left[\ln\left(x+\sqrt{x^2+1}\right)\right]^2+c$

19. $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$ is equal to

(a) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+c$

(b) $\frac{1}{\sqrt{2}}\sin^{-1}(\sin x-\cos x)+c$

(c) $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$

(d) $\frac{1}{\sqrt{2}}\sin^{-1}(\sin x+\cos x)+c$

20. $\int\frac{\sqrt{5+x^{10}}}{x^{16}}dx$ is equal to

(a) $-\frac{1}{75}\left(1+\frac{5}{x^{10}}\right)^{3/2}+c$ (b) $-\frac{1}{75}\left(1-\frac{5}{x^{10}}\right)^{3/2}+c$

(c) $\frac{1}{75}\left(1-\frac{5}{x^{10}}\right)^{3/2}+c$ (d) $\frac{1}{75}\left(1+\frac{5}{x^{10}}\right)^{3/2}+c$

21. $\int\frac{\left(\sqrt{x^2+1}\right)\{\ln(x^2+1)-2\ln x\}}{x^4}dx$ is equal to

(a) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(b) $\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(c) $\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2+3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

(d) $\frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right]+c$

22. If $I=\int\frac{dx}{5+4\cos x}=f(x)+c$, then $f(x)=$

(a) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$ (b) $-\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$

(c) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)$ (d) none of these

23. $\int e^{ax}\cos bx\,dx$ is equal to

(a) real part of $\int e^{(a+bi)x}dx$

(b) imaginary part of $\int e^{(a+bi)x}dx$

(c) neither real nor imaginary part of $\int e^{(a+bi)x}dx$

(d) none of these

24. $\int\frac{2^{x+1}-5^{x-1}}{10^x}dx$ is equal to

(a) $\frac{2}{\ln 5}5^{x}+\frac{1}{5\ln 2}2^x+c$ (b) $\frac{-2}{\ln 5}5^{-x}+\frac{1}{5\ln 2}2^{-x}+c$

(c) $\frac{1}{2\ln 5}5^{-x}-\frac{1}{5\ln 2}2^{-x}+c$

(d) none of these

25. $\int\frac{dx}{x-x^3}=A\ln\left|\frac{x^2}{1-x^2}\right|+c$, then A is equal to

(a) 2 (b) 1/2 (c) 2/3 (d) 1/3

26. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$ is equal to

(a) $\frac{x^2}{2}+\log|x|+c$ (b) $\frac{x^2}{2}+\log|x|+2x+c$

(c) $\frac{1}{3}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^3+c$ (d) $\frac{x^2}{2}+\log x-2x+c$

27. If $\int\frac{\sqrt{\cot x}}{\sin x\cos x}dx=A\sqrt{\cot x}+B$, then A is equal to

(a) 1 (b) 2 (c) –1 (d) –2

28. $I=\int\frac{(10x^9+10^x\log_e 10)}{(x^{10}+10^x)}dx$ is equal to

(a) $10^x+x^{10}+c$ (b) $10^x-x^{10}+c$

(c) $10^x+x^{10}+c$ (d) $\log_e(10^x+x^{10})+c$

29. $\int \frac{x^3-3x+7}{x^2+4}dx$ is equal to

(a) $\frac{x^2}{2}+\frac{7}{2}\ln(x^2+4)+c$

(b) $\frac{x^2}{2}+\frac{7}{2}\tan^{-1}\frac{x}{2}-\frac{7}{2}\ln(x^2+4)+c$

(c) $-\frac{x^2}{2}+\frac{7}{2}\tan^{-1}\frac{x^2}{2}+\frac{7}{2}\ln(x^2+4)+c$

(d) $\frac{x}{2}+\frac{7}{2}\tan^{-1}\frac{x}{2}+c$

Multiple Correct Answer Type

30. Suppose f is a function that satisfies the equation $f(x+y)=f(x)+f(y)+x^2y+xy^2$ for all real numbers x and y. If $\lim_{x\to 0}\frac{f(x)}{x}=1$, then

(a) $f(x)>0$ for $x>0$ and $f(x)<0$ for $x<0$

(b) $f'(0)=1$

(c) $f''(0)=1$

(d) $f'''(x)$ is a constant for all real x

31. If $\int_0^x\left(x-[x]-\frac{1}{2}\right)dx=f(x)\cdot g(x)$, where $[x]$ and $\{x\}$ are integral and fractional parts of x, respectively

(a) $f(x)=\frac{\{x\}}{2}$ (b) $g(x)=(\{x\}-1)$

(c) $f(x)=\frac{[x]}{2}$ (d) $g(x)=([x]-1)$

32. Let 'e' be the eccentricity of a hyperbola and $f(e)$ be the eccentricity of its conjugate hyperbola, then

$$\int_1^3 \underbrace{f(f(f....f(e)))}_{n\text{ times}}de=$$

(a) $2\sqrt{2}$, if n is even (b) $2\sqrt{2}$, if n is odd

(c) 4, if n is even (d) 4, if n is odd

33. If $\int\sqrt{x+\sqrt{x^2+2}}\,dx=A\left\{x+\sqrt{x^2+2}\right\}^{3/2}+\frac{B}{\sqrt{x+\sqrt{x^2+2}}}+c$, then

(a) $A=1/3$ (b) $B=-2$

(c) $A=2/3$ (d) $B=-1$

Matrix-Match Type

34. Match the following :

Column-I		Column-II	
A.	If $\int x^2d(\tan^{-1}x)dx=x-f(x)+c$, then $f(1)$ is equal to	p.	1/8
B.	If $\int x^2e^{2x}dx=e^{2x}f(x)+c$, then the minimum value of $f(x)$ is	q.	0
C.	If $\int\frac{(x^4+1)}{x(x^2+1)^2}dx=a\log\lvert x\rvert+\frac{b}{x^2+1}+c$ then $(a-b)$ is equal to	r.	$\pi/4$

35. Match the following :

Column-I		Column-II	
A.	$\int\frac{1}{1+\sin x+\cos x}dx=$	p.	$\frac{1}{2}\log\left\lvert\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right\rvert+c$
B.	$\int\frac{1}{\sqrt{3}\sin x+\cos x}dx=$	q.	$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan(x/2)}{\sqrt{3}}\right)+c$
C.	$\int\frac{1}{1-2\sin x}dx=$	r.	$\log\left\lvert 1+\tan\frac{x}{2}\right\rvert+c$
D.	$\int\frac{1}{2+\cos x}dx=$	s.	$\frac{1}{\sqrt{3}}\log\left\lvert\frac{\tan\frac{x}{2}-2-\sqrt{3}}{\tan\frac{x}{2}-2+\sqrt{3}}\right\rvert+c$

SOLUTIONS

1. **(b) :** $I=\int\frac{dx}{e^x+4e^{-x}}=\int\frac{e^xdx}{e^{2x}+4}$

Let $e^x=t\Rightarrow e^x\,dx=dt$

$$\Rightarrow I=\int\frac{dt}{t^2+4}=\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+c=\frac{1}{2}\tan^{-1}\left(\frac{e^x}{2}\right)+c.$$

2. **(b) :** Put $\sin x=A(\sin x-\cos x)+B(\cos x+\sin x)$

$\Rightarrow \sin x=(A+B)\sin x+(B-A)\cos x$

Equating the coefficients of $\sin x$ and $\cos x$, we get

$A+B=1$ and $B-A=0 \Rightarrow A=\frac{1}{2}, B=\frac{1}{2}$

$$\therefore \quad I=\int\frac{\frac{1}{2}(\sin x-\cos x)+\frac{1}{2}(\cos x+\sin x)}{\sin x-\cos x}dx$$

$$=\frac{1}{2}\int dx+\frac{1}{2}\int\frac{\cos x+\sin x}{\sin x-\cos x}dx$$

$$=\frac{x}{2}+\frac{1}{2}\log(\sin x-\cos x)+c$$

3. (c) : $I=\int \frac{2\cos\frac{9x}{2}\cos\frac{x}{2}\cos\frac{3x}{2}}{\left[1-2\left(2\cos^2\frac{3x}{2}-1\right)\right]\cos\frac{3x}{2}}dx$

$=\int \frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{3\cos\frac{3x}{2}-4\cos^3\frac{3x}{2}}dx=\int \frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{-\cos\frac{9x}{2}}dx$

$=-\int(\cos 2x+\cos x)dx=-\frac{\sin 2x}{2}-\sin x+c$

4. (b) : $I=\int \tan^2 2x\tan 2x\sec 2x\,dx$

$=\int(\sec^2 2x-1)\sec 2x\tan 2x\,dx$

Put $\sec 2x=t \Rightarrow 2\sec 2x\tan 2x\,dx=dt$

$\therefore\ I=\frac{1}{2}\int(t^2-1)dt=\frac{1}{2}\left(\frac{t^3}{3}-t\right)=\frac{1}{6}(\sec^3 2x-3\sec 2x)+c$

5. (b)

6. (c) : $I=\int \frac{\log\left(\frac{x+1}{x}\right)}{\left(\frac{x+1}{x}\right)}\cdot\frac{1}{x^2}dx$

Put $t=\frac{x+1}{x}=1+\frac{1}{x} \Rightarrow dt=-\frac{1}{x^2}dx$

$\therefore\ I=-\int(\log t)\frac{1}{t}dt=-\frac{1}{2}(\log t)^2+c$

$=-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2+c$

7. (a) : Here $I=\int \frac{1}{x\sqrt{1-x^3}}dx$. Multiply numerator and denominator by x^2.
Put $1-x^3=t^2$ $\quad\therefore -3x^2dx=2t\,dt$

$\therefore\ I=\frac{2}{3}\int\frac{dt}{t^2-1}=\frac{2}{3}\cdot\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+b$

$=\frac{1}{3}\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|+b$

8. (d)

9. (c) : $\int x^2\sin x\,dx=-x^2\cos x+2x\sin x+2\cos x+c$

10. (c)

11. (c) : $I=\int\frac{\sec^6 x}{1+\tan^6 x}dx$

$=\int\frac{(1+\tan^2 x)^2\sec^2 x\,dx}{1+\tan^6 x}$

Put $\tan x=p \Rightarrow \sec^2 x\,dx=dp$

$\therefore\ I=\int\frac{(1+p^2)^2dp}{1+p^6}=\int\frac{(1+p^2)}{p^4-p^2+1}dp=\int\frac{p^2\left(1+\frac{1}{p^2}\right)}{p^2\left(p^2+\frac{1}{p^2}-1\right)}dp$

$=\int\frac{dk}{k^2+1}=\tan^{-1}(k)+c$

$\left[\because\ \text{If } p-\frac{1}{p}=k\Rightarrow\left(1+\frac{1}{p^2}\right)dp=dk\right]$

$=\tan^{-1}\left(p-\frac{1}{p}\right)+c=\tan^{-1}(\tan x-\cot x)+c$

12. (a) : Put $x=p^6 \Rightarrow dx=6p^5\,dp$

$\therefore\ I=\int\frac{6p^5(p^6+p^4+p)}{p^6(1+p^2)}dp=\int\frac{6(p^5+p^3+1)}{(p^2+1)}dp$

$=6\int p^3\,dp+\int\frac{6}{p^2+1}dp$

$=\frac{6p^4}{4}+6\tan^{-1}p=\frac{3}{2}x^{2/3}+6\tan^{-1}(x^{1/6})+c$

13. (c) : $I=\int x\cos x\,dx+\int\sin x\cos x\,dx$

$=x\sin x+\cos x-\frac{\cos^2 x}{2}+c$

14. (b) : Let $I=\int\frac{1}{4\cos^3 x-3\cos x}dx=\int\frac{1}{\cos 3x}dx$

$=\int\sec 3x\,dx=\frac{1}{3}\ln|\sec 3x+\tan 3x|+c$

15. (d) : Let $I=\int\frac{1}{x[(\ln x)^2+4\ln x-1]}dx$

Let $\ln x=t \Rightarrow \frac{1}{x}dx=dt$

$\therefore\ I=\int\frac{dt}{t^2+4t-1}=\int\frac{dt}{(t+2)^2-(\sqrt{5})^2}$

$=\frac{1}{2\sqrt{5}}\ln\left|\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}}\right|+c=\frac{1}{2\sqrt{5}}\ln\left|\frac{\ln x+2-\sqrt{5}}{\ln x+2+\sqrt{5}}\right|+c.$

16. (a) : Put $t=\sin x-\cos x\Rightarrow t^2=1-\sin 2x$

$\therefore\ I=\int\frac{dt}{9+16(1-t^2)}=\int\frac{dt}{25-16t^2}$

$=\frac{1}{16}\int\frac{dt}{\left(\frac{5}{4}\right)^2-t^2}=\frac{1}{16}\times\frac{1}{2\left(\frac{5}{4}\right)}\ln\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|+c$

$$= \frac{1}{40}\ln\left|\frac{5+4t}{5-4t}\right| + c = \frac{1}{40}\ln\left|\frac{5+4(\sin x-\cos x)}{5-4(\sin x-\cos x)}\right| + c$$

17. (c) : Here $\int \frac{dx}{1+\sin x} = \int \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)}$

$$= \frac{1}{2}\int \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx = \frac{1}{2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-1/2} + c$$

$$= -\tan\left(\frac{\pi}{4}-\frac{x}{2}\right) + c = \tan\left(\frac{x}{2}-\frac{\pi}{4}\right) + c$$

$\Rightarrow \quad a = -\frac{\pi}{4}$, b is any arbitrary constant.

18. (c) : Put $\ln(x+\sqrt{x^2+1}) = t$

$$\Rightarrow \quad \frac{1}{x+\sqrt{x^2+1}}\left(1+\frac{x}{\sqrt{x^2+1}}\right)dx = dt$$

$$\therefore \quad I = \int t\, dt = \frac{t^2}{2} + c = \frac{1}{2}\left[\ln\left(x+\sqrt{x^2+1}\right)\right]^2 + c$$

19. (c) : Let $I = \int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

$$= \sqrt{2}\int \frac{\sin x+\cos x}{\sqrt{2\sin x\cdot\cos x}}dx = \sqrt{2}\int \frac{(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$$

Let $t = \sin x - \cos x \quad \Rightarrow \quad t^2 = 1 - \sin 2x$

$$\therefore \quad I = \sqrt{2}\int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\sin^{-1}(t) + c$$

$$= \sqrt{2}\sin^{-1}(\sin x - \cos x) + c$$

20. (a) : Let $I = \int \frac{\sqrt{5+x^{10}}}{x^{16}}dx$

Let $1+\frac{5}{x^{10}} = t \Rightarrow 5\left(\frac{-10}{x^{11}}\right)dx = dt$

$$\therefore \quad I = -\frac{1}{50}\int \sqrt{t}\,dt$$

$$= -\frac{1}{50}\times\frac{2}{3}t^{3/2} = -\frac{1}{75}\left(1+\frac{5}{x^{10}}\right)^{3/2} + c$$

21. (d) : Let $I = \int \frac{\left(\sqrt{\frac{x^2+1}{x^2}}\right)\ln\left(\frac{x^2+1}{x^2}\right)}{x^3}dx$

Put $\frac{x^2+1}{x^2} = t \Rightarrow -\frac{2}{x^3}dx = dt$

$$\therefore \quad I = -\frac{1}{2}\int \sqrt{t}\ln t\, dt$$

$$= -\frac{1}{2}\left[(\ln t).\frac{2t^{3/2}}{3} - \frac{2}{3}\int \frac{1}{t}.t^{3/2}dt\right] = \frac{1}{9}t^{3/2}[2-3\ln t] + c$$

$$= \frac{1}{9}\frac{(x^2+1)\sqrt{x^2+1}}{x^3}\left[2-3\ln\left(\frac{x^2+1}{x^2}\right)\right] + c$$

22. (a) : Put $t = \tan\frac{x}{2} \Rightarrow dx = \frac{2dt}{1+t^2}$

$$\therefore \quad I = \int \frac{\frac{2dt}{1+t^2}}{5+4\left(\frac{1-t^2}{1+t^2}\right)} = 2\int \frac{dt}{t^2+9} = 2\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right) + c$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right) + c$$

23. (a) : Let $I = \int e^{ax}\cos bx\, dx$

$= \text{real part of} \int e^{ax}\, e^{ibx}dx = \text{real part of} \int e^{(a+ib)x}dx$

24. (b) : $\int \frac{2^{x+1}-5^{x-1}}{10^x}dx = \int\left[2\left(\frac{1}{5}\right)^x - \frac{1}{5}\left(\frac{1}{2}\right)^x\right]dx$

$$= \frac{2\left(\frac{1}{5}\right)^x}{\ln\frac{1}{5}} - \frac{\frac{1}{5}\left(\frac{1}{2}\right)^x}{\ln\frac{1}{2}} + c = \frac{-2}{\ln 5}5^{-x} + \frac{1}{5\ln 2}2^{-x} + c$$

25. (b) : Let $I = \int \frac{dx}{x-x^3} = \int \frac{dx}{x^3\left(\frac{1}{x^2}-1\right)}$

Put $\frac{1}{x^2} = t \quad \Rightarrow \quad -\frac{2}{x^3}dx = dt$

$$\therefore \quad I = -\frac{1}{2}\ln|t-1| + c = -\frac{1}{2}\ln\left|\frac{1-x^2}{x^2}\right| + c$$

$$= \frac{1}{2}\ln\left|\frac{x^2}{1-x^2}\right| + c \Rightarrow A = \frac{1}{2}$$

26. (d) : $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx = \int\left(x+\frac{1}{x}-2\right)dx$

$$= \frac{x^2}{2} + \ln|x| - 2x + c$$

27. (d) : $\int \frac{\sqrt{\cot x}}{\sin x\cos x}dx = \int \frac{\sqrt{\cot x}}{\cot x}\cdot \text{cosec}^2 x dx$

$$= \int \frac{\text{cosec}^2 x}{\sqrt{\cot x}}dx = -2\sqrt{\cot x} + B$$

28. (d) : Put $10^x + x^{10} = p$

$\Rightarrow \quad (10^x \log 10 + 10x^9)dx = dp$

$\therefore \quad I = \log(x^{10} + 10^x) + c$

29. (b) : $\dfrac{x^3 - 3x + 7}{x^2 + 4} = x - \dfrac{7(x-1)}{x^2+4}$

$$\therefore \quad \int \frac{x^3 - 3x + 7}{x^2 + 4} dx = \frac{x^2}{2} - 7\int \frac{(x-1)}{x^2+4} dx$$

$$= \frac{x^2}{2} + \frac{7}{2}\tan^{-1}\frac{x}{2} - \frac{7}{2}\ln(x^2+4) + c$$

30. (a, b, d) : Observe that $f(0) = 0$ and

$$f'(0) = \lim_{h \to 0} \frac{f(h)}{h} = 1$$

Also $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{f(h) + x^2 h + xh^2}{h} = x^2 + 1$$

Hence, $f(x) = \dfrac{x^3}{3} + x$

31. (a, b) : Here $I = \displaystyle\int_0^{[x]+\{x\}} \left(\{x\} - \frac{1}{2}\right) dx$

$$= \int_0^{[x]} \left(\{x\} - \frac{1}{2}\right) dx + \int_{[x]}^{[x]+\{x\}} \left(\{x\} - \frac{1}{2}\right) dx$$

$$= [x]\int_0^1 \left(\{x\} - \frac{1}{2}\right) dx + \int_0^{\{x\}} \left(\{x\} - \frac{1}{2}\right) dx$$

$$= [x]\int_0^1 \left(x - \frac{1}{2}\right) dx + \int_0^{\{x\}} \left(x - \frac{1}{2}\right) dx$$

$$= [x]\left(\frac{x^2}{2} - \frac{x}{2}\right)_0^1 + \left(\frac{x^2}{2} - \frac{x}{2}\right)_0^{\{x\}}$$

$$= 0 + \frac{\{x\}(\{x\} - 1)}{2} = \frac{\{x\}}{2}(\{x\} - 1)$$

32. (b, c) : Let e_1, e_2 are eccentricities of hyperbola and its conjugate, then

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1 \Rightarrow \frac{1}{e_2^2} = \frac{e_1^2 - 1}{e_1^2} \Rightarrow e_2 = \frac{e_1}{\sqrt{e_1^2 - 1}}$$

$$\therefore \quad f(e) = \frac{e}{\sqrt{e^2 - 1}}$$

Now, $f(f(e)) = f\left(\dfrac{e}{\sqrt{e^2-1}}\right) = \dfrac{\dfrac{e}{\sqrt{e^2-1}}}{\sqrt{\dfrac{e^2}{e^2-1} - 1}} = e$

$$\therefore \quad \underbrace{f(f(f \ldots f(e)))}_{(n \text{ times})} = \begin{cases} \dfrac{e}{\sqrt{e^2-1}}, & \text{if } n \text{ is odd} \\ e \quad, & \text{if } n \text{ is even} \end{cases}$$

33. (a, b) : Put $x + \sqrt{x^2 + 2} = t \Rightarrow x^2 + 2 = t^2 + x^2 - 2tx$

$$\Rightarrow \quad x = \frac{1}{2}\left(t - \frac{2}{t}\right)$$

$$\therefore \quad \int \sqrt{x + \sqrt{x^2+2}}\, dx = \frac{1}{2}\int t^{1/2}\left(1 + \frac{2}{t^2}\right) dt$$

$$= \frac{1}{2}\int t^{1/2} dt + \int t^{-3/2} dt = \frac{1}{3}t^{3/2} - \frac{2}{\sqrt{t}} + c$$

34. **A → r; B → p; C → q**

(A) $\displaystyle\int x^2 \cdot \frac{1}{1+x^2} dx = x - \tan^{-1} x + c$

$$\Rightarrow f(x) = \tan^{-1} x \Rightarrow f(1) = \tan^{-1} 1 = \frac{\pi}{4}$$

(B) $\displaystyle\int x^2 e^{2x} dx = \frac{1}{2}x^2 \cdot e^{2x} - \int 2x \cdot \frac{1}{2}e^{2x} dx$

$$= \frac{1}{2}x^2 e^{2x} - \left[x \cdot \frac{1}{2}e^{2x} - \frac{1}{4}e^{2x}\right] = \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right)e^{2x} + c$$

(C) $\displaystyle\int \frac{(x^4+1)}{x(x^2+1)^2} dx = \int \frac{((x^2+1)^2 - 2x^2)}{x(x^2+1)^2} dx$

$$= \int \frac{1}{x} dx - 2\int \frac{x\,dx}{(x^2+1)^2} = \log|x| + \frac{1}{x^2+1} + c$$

35. **A → r; B → p; C → s; D → q**

◆◆

PUZZLE CORNER

SOLUTION · DECEMBER 2018

2– 2	2– 1	3	60× 4	5	9+ 6
4	11+ 5	6 6	60× 2	3	1
18× 3	4	5× 1	5	6	2
6	2	5	1	60× 4	3
1	9+ 3	4	2+ 6	2 2	5
30× 5	6	2	3	4+ 1	4

CLASS XII **Series 9**

CBSE DRILL

Synopsis and Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

Linear Programming / Probability

Linear Programming

LINEAR PROGRAMMING PROBLEM (L.P.P.)

Linear programming is a method for finding the optimal values (maximum or minimum) of quantities subject to the constraints expressed as linear inequation or inequalities.

TERMS USED IN L.P.P.

- **Objective function :** The linear function which is to be maximised or minimised under given constraints.
- **Decision variables :** The variables x and y in the objective function $Z = ax + by$ are called decision variables.
- **Constraints :** The restrictions or linear inequalities or inequations on the variables of a L.P.P. which describe the conditions under which the optimisation is to be accomplish. The conditions $x \geq 0$, $y \geq 0$ are called non-negative constraints.
- **Feasible region :** The common region determined by all the constraints including non-negative restrictions of a L.P.P.
- **Bounded and unbounded feasible region :** A feasible region of a system of linear inequations is said to be bounded if it can be enclosed within a circle. But if the feasible region extends indefinitely in any direction, then the feasible region is called unbounded.
- **Corner point :** A point in the feasible region which is the intersection of two boundary lines is called corner point.
- **Feasible solution :** A feasible solution is the set of points, whose co-ordinates satisfy the constraints and non-negativity of a L.P.P.
- **Optimal solution :** A feasible solution of L.P.P. which maximizes or minimizes the objective function.

MATHEMATICAL FORMULATION OF L.P.P.

The procedure for mathematical formulation of a linear programming problem consists of the following steps :

I. In every L.P.P. certain decisions are to be made. These decisions are represented by decision variables. These decision variables are those quantities whose values are to be determined. Identify the variables and denote them by $x_1, x_2, x_3, \ldots$

II. Identify the objective function and express it as a linear function of the variables introduced in step I.

III. In a L.P.P., the objective function may be in the form of maximising profits or minimising costs. So, after expressing the objective functions as a linear function of the decision variables, we must find the type of optimisation *i.e.*, maximisation or minimisation. Identify the type of the objective function.

IV. Identify the set of the constraints, stated in terms of decision variables and express them as linear inequations or equations as the case may be.

SOME IMPORTANT THEOREMS OF L.P.P.

- Let R be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at the corner point (vertex) of the feasible region.
- Let R be the feasible region for a linear programming problem, and let $Z = ax + by$ be the objective function. If R is bounded, then the objective function Z has both a maximum and minimum value on R and each of these occurs at a corner point (vertex) of R.
  Note : If R is unbounded, then a maximum or minimum value of the objective function may not exist. However, if it exists, it must occur at a corner point of R.

GRAPHICAL METHOD OF SOLVING L.P.P.

Corner Point Method

To solve a linear programming problem by corner point method, we follow the following steps :

(i) Formulate the given L.P.P. in mathematical form.

(ii) Convert all inequations into equations and draw their graphs.

(iii) Determine the region represented by each inequation.

(iv) Obtain the region in xy-plane containing all points that simultaneously satisfy all constraints including non-negativity restrictions. The region so obtained is the feasible region.

(v) Determine the coordinates of the vertices (corner points) of the feasible region obtained in Step (iv). These vertices are known as the extreme points of the set of all feasible solutions of the L.P.P.

(vi) Obtain the values of the objective function at each of the vertices and let M and m are the maximum and minimum values respectively at the vertices.

(vii) If feasible region is bounded then M and m are the maximum and minimum values of objective function.

(viii) If feasible region is unbounded then

- M is maximum value of $Z = ax + by$ if open half plane determined by $ax + by > M$ has no point in common with feasible region otherwise no maximum value.
- m is minimum value of $Z = ax + by$ if open half plane determined by $ax + by < m$ has no point in common with feasible region otherwise no minimum value.

DIFFERENT TYPES OF L.P.P.

Following are the important types of linear programming problems:

- **Diet problems :** In diet problems, amounts of different kinds of nutrients/constituents which should be included in a diet are determined so as to minimize the cost of the desired diet such that it contains a certain minimum amount of each nutrient/constituent.
- **Manufacturing problems (Optimal product line problems) :** In such type of problems, the number of units of different products which is to be produced and sold by a firm when each product requires a fixed man power, machine hour, labour hour per unit of product, warehouse space per unit of the output etc., are to be determined so as to achieve maximum profit.
- **Transportation problems :** In such type of problems, a transportation schedule is determined so as to find the cheapest way of transporting a product from plants/factories situated at different locations to different markets are to be determined so as to minimize the cost of transportation subject to limitations (constraints) of demand of each market and supply from each plant or factory.

Probability

CONDITIONAL PROBABILITY

Let A and B be any two events associated with a random experiment. The probability of occurrence of event A when the event B has already occurred is called conditional probability and it is denoted by $P(A/B)$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

MtG

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Properties of Conditional Probability

For events A, B, C and sample space S,

- $P(S/A) = P(A/A) = 1$
- $0 \le P(A/B) \le 1, P(B) \ne 0$
- $P(A'/B) = 1 - P(A/B), P(B) \ne 0$
- $P((A \cup B)/C) = P(A/C) + P(B/C) - P((A \cap B)/C), P(C) \ne 0$

MULTIPLICATION THEOREM ON PROBABILITY

If A and B are two events associated with a random experiment, then

$P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B), P(A) \ne 0, P(B) \ne 0.$

In general, if $A_1, A_2, A_3, \ldots, A_n$ are n events such that $A_1 \ne \phi, A_1 A_2 \ne \phi, A_1 A_2 A_3 \ne \phi, \ldots, A_1 A_2 \ldots A_{n-1} \ne \phi$, then $P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) \times P(A_2/A_1) \times P(A_3/(A_1 \cap A_2)) \times \ldots \times P(A_n/(A_1 \cap A_2 \cap \ldots \cap A_{n-1}))$

INDEPENDENT EVENTS

Two events A and B are said to be independent events, if the happening (or non happening) of any random experiment does not affect the happening (or non-happening) of the other *i.e.*, $P(A \cap B) = P(A) \cdot P(B)$.

Note :

- Two events A and B are said to be dependent if they are not independent, *i.e.*, if $P(A \cap B) \ne P(A) \cdot P(B)$
- Three events A, B and C are said to be mutually independent, if
  $P(A \cap B) = P(A) \cdot P(B), P(A \cap C) = P(A) \cdot P(C)$
  $P(B \cap C) = P(B) \cdot P(C)$
  and $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$
  If at least one of the above is not true for three given events, we say that the events are not independent.

THEOREM OF TOTAL PROBABILITY

Let S be the sample space and $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event of non-zero probability which occurs with E_1 or E_2 or … or E_n, then

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \ldots + P(E_n)P(A/E_n) = \sum_{i=1}^{n} P(E_i)P(A/E_i)$$

BAYES' THEOREM

If $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events $P(E_i) \ne 0$ $(i = 1, 2, \ldots, n)$, then for any event A which is a subset of $\bigcup_{i=1}^{n} E_i$ such that $P(A) > 0$, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\,P(A/E_i)}, i = 1, 2, \ldots, n$$

RANDOM VARIABLE AND ITS PROBABILITY DISTRIBUTION

Random Variable : Let S be the sample space associated with a given random experiment. Then a random variable is a real valued function whose domain is a subset of sample space of the experiment.

Probability Distribution : If a random variable X takes value $x_1, x_2, x_3, \ldots, x_n$ with respective probabilities $p_1, p_2, \ldots, p_n$, then

X	:	x_1	x_2	…	x_n
$P(x)$	:	p_1	p_2	…	p_n

is known as the probability distribution of x.

Mean, Variance, Standard Deviation of a Random Variable

- Mean $= E(X) = \mu = p_1 x_1 + p_2 x_2 + \ldots + p_n x_n = \sum_{i=1}^{n} p_i x_i$
- $\text{Var}(X) = \sum_{i=1}^{n} (x_i - \mu)^2 p_i$ or $\sum_{i=1}^{n} p_i x_i^2 - \left(\sum_{i=1}^{n} p_i x_i\right)^2$
  $= E(X^2) - [E(X)]^2$
- Standard Deviation, $\sigma_x = \sqrt{\text{Var}(X)}$

BERNOULLI TRIALS AND BINOMIAL DISTRIBUTION

Bernoulli Trials : Set of n trails are said to be Bernoulli trials if

- The value of n is finite.
- Each and every trial is independent.
- Each trial has only two outcomes : success or failure.
- Probability of success or failure for each trial is fixed.

Binomial Distribution : A random variable X which takes values $0, 1, 2, \ldots, n$ follows binomial distribution if its probability distribution is given by

$P(x = r) = {}^nC_r p^r q^{n-r}, r = 0, 1, 2, \ldots, n$ where $p + q = 1, p, q > 0$,
p = probability of success, q = probability of failure,
n = number of trials.

Mean, Variance and Standard Deviation

- Mean $= np$
- Variance $= npq$
- Standard Deviation $= \sqrt{npq}$

WORK IT OUT

VERY SHORT ANSWER TYPE

1. A pair of dice is thrown 200 times. If getting a sum of 9 is considered a success, find the mean of the number of successes.

2. Find the maximum value of $Z = x + y$ subject to constraints $x + 2y \leq 70$, $2x + y \leq 95$, $x \geq 0$, $y \geq 0$.

3. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = px + qy$, where $p, q > 0$. Find the condition on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20).

4. A bag contains 10 white and 6 black balls. 4 balls are successively drawn out without replacement. What is the probability that they are alternately of different colours?

5. Let X denotes the number of hours a student study during a randomly selected school day. The probability that X can take the value x has the following form, where k is some unknown constant.

$$P(X = x) = \begin{cases} 0.1 & ,\text{if } x = 0 \\ kx & ,\text{if } x = 1 \text{ or } 2 \\ k(5-x) & ,\text{if } x = 3 \text{ or } 4 \\ 0 & ,\text{otherwise}. \end{cases}$$

Find the value of k.

SHORT ANSWER TYPE

6. Assume that on an average, one telephone out of 10 is busy. Six telephone numbers are randomly selected and called. Find the probability that four of them will be busy.

7. If A and B are two events associated with the same random experiment such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$, then find $P(B)$.

8. Three events A, B and C have probabilities $\frac{2}{5}, \frac{1}{3}$ and $\frac{1}{2}$ respectively. Given that $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$, find the value of $P(C/B)$ and $P(\bar{A} \cap \bar{C})$.

9. Two thirds of the students in a class are boys and the rest girls. It is known that the probability of a girl getting a first class is 0.25 and that of a boy getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

10. Consider the objective function, $Z = 4x + 6y$ subject to $3x + 2y \leq 12$, $x + y \geq 4$, $x \geq 0$, $y \geq 0$. If $p = Z_{\max} - Z_{\min}$. Then find p.

LONG ANSWER TYPE - I

11. Solve the following L.P.P by graphical method :
Minimize $Z = 20x + 10y$ subject to constraints :
$x + 2y \leq 40$, $3x + y \geq 30$, $4x + 3y \geq 60$ and $x, y \geq 0$.

12. A shopkeeper sells three types of seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4 : 4 : 2 respectively. The germination rates of three types of seeds are 45%, 60% and 35%. Calculate the probability
    - (i) that it will not germinate given that the seed is of type A_3.
    - (ii) of a randomly chosen seed to germinate.
    - (iii) that it is of type A_2 given that a randomly chosen seed does not germinate.

13. The sum and the product of the mean and variance of a binomial distribution are 24 and 128 respectively. Find the distribution.

14. There is a factory located at each of the two places P and Q. From these locations, a certain commodity is delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are 8 and 6 units respectively. The cost of transportation per unit is shown :

Transportation Cost (in ₹)			
From \ To	A	B	C
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. Formulate the above as a linear programming problem.

15. A, B and C are three events associated with a random experiment. If the probability of occurring exactly one event out of A and B is $1 - x$, out of B and C is $1 - 2x$, out of C and A is $1 - x$ and the probability of occurring all the three events simultaneously is x^2, then prove that the probability that atleast one out of A, B and C will occur is greater than 1/2.

EXAM ALERT 2019

Exam	Date
JEE Main I	6th to 20th January
JEE Main II	6th to 20th April
VITEEE	10th to 21st April
SRMJEEE	15th to 25th April
AMU (Engg.)	28th April
COMEDK (Engg.)	12th May
JEE Advanced	19th May

CONCEPT MAP

Class XI

Representation of Sets

- **Set Builder Form :** A set is described by characterising property $P(x)$ of its element x.
- **Roster or Tabular Form :** A set is described by listing all the elements separated by commas.

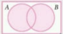
If A, B and C are finite sets, U be the finite universal set, then

- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B) = n(A) + n(B)$, if A, B are disjoint.
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
- $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$
- $n(\text{exactly two}) = n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- $n(\text{exactly one}) = n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$

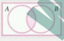
- **Singleton set :** A set with only one element.
- **Empty set :** A set with no element.
- **Finite set :** A set contains finite number of elements.
- **Infinite set :** Set which is not finite.
- **Equal sets :** Two sets A and B are equal if they have exactly same elements.
- **Equivalent sets :** Two finite sets A and B are equivalent if they have same number of elements.
- **Universal set :** The set that contains all the sets in the given context.
- **Subset :** A set A is said to be subset of set B if each element of set A is also an element of set B *i.e.*, $A \subseteq B$.
- **Proper subset :** If set A is a subset of set B but $A \neq B$, then $A \subset B$.
- **Superset :** If set A is a subset of set B *i.e.*, $A \subseteq B$, then set B is called superset.
- **Power set :** Set of all subsets of a given set.

Laws of Algebra of Sets

- **Idempotent :** $A \cup A = A; A \cap A = A$
- **Commutative :** $A \cup B = B \cup A; A \cap B = B \cap A$
- **Associative :** $A \cup (B \cup C) = (A \cup B) \cup C$
  $A \cap (B \cap C) = (A \cap B) \cap C$
- **Identity :** $A \cup \phi = \phi \cup A = A, A \cap \phi = \phi$
  $A \cap U = U \cap A = A, A \cup U = U$
- **Distributive :**
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

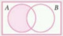


Union : It consists of all the elements of set A and set B with common element taken once *i.e.*, $(A \cup B) = \{x : x \in A \text{ or } x \in B\}$

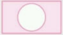


Intersection : It consists of those elements which belong to both set A and B *i.e.*, $(A \cap B) = \{x : x \in A \text{ and } x \in B\}$

- **Complement Law :** $A \cup A' = U; A \cap A' = \phi$
- **De Morgan's Law :** $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$
- **Law of Double Complement :** $(A')' = A$
- **Law of Empty Set and Universal Set :** $\phi' = U$ and U



Complement : Set of all elements of universal set U which are not the elements of set A *i.e.*, $A' = U - A$.



Difference : Set of elements which belong to set A but not to set B *i.e.*, $A - B = \{x : x \in A, x \notin B\}$

INDEFINITE INTEGRATION

CONCEPT MAP

Class XII

- $\int dx = x + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int \frac{1}{x} dx = \log_e |x| + C$
- $\int \cos x dx = \sin x + C$
- $\int \text{cosec}^2 x dx = -\cot x + C$
- $\int \sec x \tan x dx = \sec x + C$
- $\int \text{cosec}\, x \cot x dx = -\text{cosec}\, x + C$
- $\int \tan x dx = \log|\sec x| + C = -\log|\cos x| + C$
- $\int \cot x dx = \log|\sin x| + C$
- $\int \sec x dx = \log|\sec x + \tan x| + C$
  $= \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$
- $\int \text{cosec}\, x dx = \log|\text{cosec}\, x - \cot x| + C$
  $= \log\left|\tan\frac{x}{2}\right| + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\log_e a} + C, a > 0$
- $\int \sin x dx = -\cos x + C$
- $\int \sec^2 x dx = \tan x + C$

- $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$
- $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{a^2 + x^2}\right| + C$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + C$
- $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2}$
  $+ \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$
- $\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2}$
  $+ \frac{a^2}{2}\log\left|x + \sqrt{a^2 + x^2}\right| + C$
- $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2}$
  $- \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

Properties

Methods of Solving Integration

Substitution

The given integral $\int f(x)dx$ can be transformed into another form by changing the independent variable x to t by substituting $x = g(t)$.

Some Substitutions

- In integration of the type $\int \frac{px+q}{ax^2+bx+c}dx$, $\frac{\int (px+q)}{\sqrt{ax^2+bx+c}}dx$ and $\int (px+q)\sqrt{ax^2+bx+c}\,dx$
  put, $px + q = A$(differential coefficient of denominator) $+ B$
- In integration of the type $\frac{ax^2+bx+c}{(dx+e)\sqrt{f(x)}}$,
  put $ax^2 + bx + c = A_1(dx+e)f'(x) + B_1(dx+e) + k$

Partial Fractions

- If deg $f(x) \geq$ deg $g(x)$, then we divide $f(x)$ by $g(x)$
  $\therefore \quad \frac{f(x)}{g(x)} = \text{Quotient} + \frac{p(x)}{g(x)}$, where deg $p(x) <$ deg $g(x)$
  Now, $\int \frac{p(x)}{g(x)}dx$ (proper fraction) is evaluated by using partial fraction decomposition.

By Parts

If u and v are two differentiable functions of x, then
$\int (uv)dx = u\int v dx - \int u'(x)\left(\int v dx\right)dx$
General preference for first function = ILATE (Inverse, Logarithmic, Algebraic, Trigonometric, Exponential)

______	___ ___
___	___ ____
________	___ ___ ___
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LONG ANSWER TYPE - II

16. A fruit grower can use two types of fertilizers in his garden, brand *P* and brand *Q*. The amounts (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs atleast 240 kg of phosphoric acid, atleast 270 kg of potash and atmost 310 kg of chlorine.

	Brand *P*	Brand *Q*
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

(i) If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used?

(ii) If the grower wants to maximize the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

17. Urn *A* contains 2 white, 1 black and 3 red balls, urn *B* contains 3 white, 2 black and 4 red balls and urn *C* contains 4 white, 3 black and 2 red balls. One urn is chosen at random and 2 balls are drawn at random from the urn. If the chosen balls happen to be red and black, what is the probability that both balls come from urn *B*?

18. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹ 5760 to invest and has space for at most 20 items. A fan and sewing machine cost ₹ 360 and ₹ 240 respectively. He can sell a fan at a profit of ₹ 22 and sewing machine at a profit of ₹ 18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximize his profit? Translate the problem into L.P.P. and solve it graphically.

19. A bag contains $(2n + 1)$ coins. It is known that n of these coins have a head on both sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is 31/42, determine the value of n.

20. A biased die is such that $P(4) = 1/10$ and other scores being equally likely. The die is tossed twice. If X is the 'number of fours seens', find the variance of the random variable X.

SOLUTIONS

1. Let p denotes probability of getting a sum of 9 on a pair of dice

Then, $p = \frac{4}{36} = \frac{1}{9}$

Now, mean $= np = 200 \times \frac{1}{9} = \frac{200}{9}$ $\qquad [\because n = 200]$

2. We have, $x + 2y \le 70$ and $2x + y \le 95$

$\Rightarrow 3x + 3y \le 70 + 95 \Rightarrow x + y \le 55$

$\therefore$ Maximum value of $(x + y)$ is 55.

3. Value of $Z = px + qy$ at (15, 15) is $15p + 15q$ and that at (0, 20) is $20q$. According to given condition, we have $15p + 15q = 20q \Rightarrow 15p = 5q \Rightarrow q = 3p$

4. Required probability $= P(BWBW) + P(WBWB)$

$= \frac{6}{16}\cdot\frac{10}{15}\cdot\frac{5}{14}\cdot\frac{9}{13} + \frac{10}{16}\cdot\frac{6}{15}\cdot\frac{9}{14}\cdot\frac{5}{13} = \frac{90}{728} = \frac{45}{364}$.

5. The probability distribution of X is

X	0	1	2	3	4
$P(X)$	0.1	k	$2k$	$2k$	k

$\because P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$

$\Rightarrow 0.1 + k + 2k + 2k + k = 1 \Rightarrow 6k = 0.9 \Rightarrow k = 0.15$

6. Let A be the event that telephone is dialled and it is busy, then $p = P(A) = \frac{1}{10}$, $q = \frac{9}{10}$ and $n = 6$.

Thus, we have a binomial distribution with $n = 6$, $p = \frac{1}{10}$ and $q = \frac{9}{10}$.

Hence, the required probability $= P(4) = {}^6C_4 p^4 q^2$

$= {}^6C_4\left(\frac{1}{10}\right)^4\left(\frac{9}{10}\right)^2 = 0.001215$

7. Given, $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{2}{3}$

Now, $P(\bar{A}) = \frac{2}{3} \Rightarrow 1 - P(A) = \frac{2}{3} \Rightarrow P(A) = \frac{1}{3}$.

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4} \Rightarrow P(B) = \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = \frac{2}{3}$.

8. We have, $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{2}$,

Also, $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$

$\therefore P(C/B) = \frac{P(C \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$

and, $P(\bar{A} \cap \bar{C}) = P(\overline{A \cup C})$

$= 1 - \{P(A) + P(C) - P(A \cap C)\} = 1 - \left(\frac{2}{5} + \frac{1}{2} - \frac{1}{5}\right) = \frac{3}{10}$

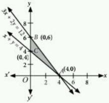
9. Let E_1, E_2 and A be the events defined as follows: E_1 = a boy is chosen from the class, E_2 = a girl is chosen from the class and, A = the student gets first class marks. Then, $P(E_1) = 2/3$, $P(E_2) = 1/3$, $P(A/E_1) = 0.28$ and $P(A/E_2) = 0.25$.

Thus, $P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) = 0.27$

10. Convert inequations into equations and draw the corresponding lines $3x + 2y = 12$, $x + y = 4$

i.e., $\frac{x}{4}+\frac{y}{6}=1$,

$\frac{x}{4}+\frac{y}{4}=1$.



As $x \geq 0$, $y \geq 0$, solution lies in first quadrant. Corner points of feasible region are $A(4, 0)$, $B(0, 6)$ and $C(0, 4)$.

Now, $Z = 4x + 6y$

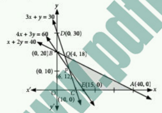
$Z(A) = 4(4) + 6(0) = 16$, $Z(B) = 4(0) + 6(6) = 36$, $Z(C) = 4(0) + 6(4) = 24$

$\therefore\ p = Z_{\max} - Z_{\min} = 36 - 16 = 20$

11. We first draw the lines

$x + 2y = 40$, $3x + y = 30$, $4x + 3y = 60$

The feasible region (shown shaded) is the bounded region $EAQPE$.



The vertices of the feasible region are

$E(15, 0)$, $A(40, 0)$, $Q(4, 18)$ and $P(6, 12)$

These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.

Given, $Z = 20x + 10y$

At $E(15, 0)$, $Z = 20 \times 15 + 10 \times 0 = 300$

At $A(40, 0)$, $Z = 20 \times 40 + 10 \times 0 = 800$

At $Q(4, 18)$, $Z = 20 \times 4 + 10 \times 18 = 260$

At $P(6, 12)$, $Z = 20 \times 6 + 10 \times 12 = 240$

Clearly, Z is minimum at $P(6, 12)$. Hence, $x = 6$, $y = 12$ is the optimal solution of the given L.P.P. The optimal value of Z is 240.

12. Consider the following events:

E_i = Seed chosen is of type A_i, $i = 1, 2, 3$

A = Seed chosen germinates.

We have, $P(E_1) = \frac{4}{10}$, $P(E_2) = \frac{4}{10}$ and $P(E_3) = \frac{2}{10}$

$$P(A/E_1) = \frac{45}{100},\ P(A/E_2) = \frac{60}{100},\ P(A/E_3) = \frac{35}{100}$$

(i) Required probability $= P(\bar{A}/E_3) = 1 - P(A/E_3)$

$= 1 - \frac{35}{100} = \frac{65}{100} = 0.65$

(ii) Required probability $= P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$

$$= \frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}\times\frac{35}{100}=\frac{490}{1000}=0.49$$

(iii) Required probability $= P(E_2/\bar{A})$

$$= \frac{P(E_2 \cap \bar{A})}{P(\bar{A})} = \frac{P(E_2)P(\bar{A}/E_2)}{P(\bar{A})} = \frac{P(E_2)(1-P(A/E_2))}{1-P(A)}$$

$$= \frac{\frac{4}{10}\times\left(1-\frac{60}{100}\right)}{1-\frac{49}{100}} = \frac{\frac{4}{10}\times\frac{40}{100}}{\frac{51}{100}} = \frac{16}{51}$$

13. According to question, we have

$np + npq = 24 \Rightarrow np(1 + q) = 24$ ...(i)

and $np\cdot npq = 128 \Rightarrow n^2p^2q = 128$ ...(ii)

Dividing the square of (i) by (ii), we get

$$\frac{n^2p^2(1+q)^2}{n^2p^2q} = \frac{24\times 24}{128} \Rightarrow \frac{1+2q+q^2}{q} = \frac{9}{2}$$

$\Rightarrow 2 + 4q + 2q^2 = 9q \Rightarrow (2q - 1)(q - 2) = 0$

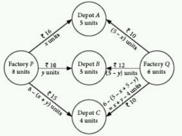
$\Rightarrow q = \frac{1}{2}, 2$ but $q \neq 2 \Rightarrow q = \frac{1}{2}$.

$\therefore\ p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$.

From (i), we get $n\cdot\frac{1}{2}\left(1+\frac{1}{2}\right) = 24 \Rightarrow n = 32$.

Hence, the binomial distribution is $\left(\frac{1}{2}+\frac{1}{2}\right)^{32}$.

14. The given information can be represented diagrammatically as shown:



Let the factory at P transports x units of commodity to depot at A and y units to depot at B. Therefore, $(8 - x - y)$ units will be transported to depot at C.

$\therefore\ x \geq 0, y \geq 0$ and $8 - x - y \geq 0 \Rightarrow x + y \leq 8$

Since the weekly requirement of the depot at A is 5 units. Therefore, the remaining $(5 - x)$ units are to be transported from the factory at Q. Similarly, $(5 - y)$ units of the commodity will be transported from the factory at Q to the depot at B and $6 - (5 - x + 5 - y) = x + y - 4$ units will be transported to the depot at C.

$\therefore\ 5 - x \geq 0, 5 - y \geq 0$ and $x + y - 4 \geq 0$

i.e., $x \leq 5, y \leq 5$ and $x + y \geq 4$.

By, the given table, total transportation cost Z is given by

$$Z = 16x + 10y + 15(8 - x - y) + 10(5 - x) + 12(5 - y) + 10(x + y - 4) = x - 7y + 190$$

Hence, the above L.P.P. can be stated mathematically as :

Minimize $Z = x - 7y + 190$,

subject to constraints :

$x + y \leq 8, x + y \geq 4, x \leq 5, y \leq 5$ and $x \geq 0, y \geq 0$.

15. Given, $P(A - B) + P(B - A) = 1 - x$

$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B) = 1 - x$

$\Rightarrow P(A) + P(B) - 2P(A \cap B) = 1 - x$ ...(i)

Similarly, $P(B) + P(C) - 2P(B \cap C) = 1 - 2x$ ...(ii)

and $P(C) + P(A) - 2P(C \cap A) = 1 - x$ ...(iii)

Adding (i), (ii) and (iii), we get

$$2(P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)) = 3 - 4x$$

$$\Rightarrow P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)) = \frac{3-4x}{2} \quad \text{...(iv)}$$

Also, $P(A \cap B \cap C) = x^2$ (given) ...(v)

Now, probability that atleast one of A, B and C will occur

$$= P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= \frac{3-4x}{2} + x^2 \quad \text{(using (iv) and (v))}$$

$$= x^2 - 2x + \frac{3}{2} = (x-1)^2 + \frac{1}{2} > \frac{1}{2} \quad (\because\ (x-1)^2 > 0)$$

16. Let x bags of brand P and y bags of brand Q of fertilizers be added to the garden, the problem can be, formulated as an L.P.P. as follows:

Optimal function $Z = 3x + 3.5y$

Subject to the constraints

$x + 2y \geq 240$

$3x + 1.5y \geq 270$

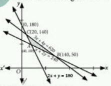
i.e. $6x + 3y \geq 540$ *i.e.* $2x + y \geq 180$,

$1.5x + 2y \leq 310$

i.e. $3x + 4y \leq 620$

$x \geq 0, y \geq 0$

Drawing the lines $x + 2y = 240$, $2x + y = 180$ and $3x + 4y = 620$, and shading the region satisfied by the inequations, we get ABC as the feasible region which is convex and bounded.



The corner points are $A(40, 100)$, $B(140, 50)$ and $C(20, 140)$. The values of $Z = 3x + 3.5y$ at the corner points are:

At $A(40, 100)$, $Z = 3 \times 40 + 3.5 \times 100 = 470$

At $B(140, 50)$, $Z = 3 \times 140 + 3.5 \times 50 = 595$

At $C(20, 140)$, $Z = 3 \times 20 + 3.5 \times 140 = 550$

(i) Minimum value of $Z = 470$ (kg of nitrogen), when 40 bags of brand P and 100 bags of brand Q of fertilizers are added to the garden.

(ii) Maximum value of $Z = 595$ (kg of nitrogen), when 140 bags of brand P and 50 bags of brand Q of fertilizers are added to the garden.

17. Let E_1, E_2, E_3 denotes the events that Urn A, Urn B, Urn C is chosen and A be the event that two balls drawn at random are red and black.

$\therefore\ P(E_1) = 1/3 = P(E_2) = P(E_3)$

If E_1 has already occured, then urn A has been chosen. The urn A contains 2 white, 1 black and 3 red balls.

$\therefore$ Probability of drawing a red and a black ball *i.e.*, $P(A/E_1)$ is $\dfrac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \dfrac{1}{5}$.

Similarly, $P(A/E_2) = \dfrac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \dfrac{2}{9}$,

$$P(A/E_3)=\frac{{}^2C_1\times{}^3C_1}{{}^9C_2}=\frac{1}{6}$$

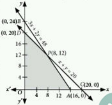
We are required to find $P(E_2/A)$. By Bayes' theorem,

$$P(E_2/A)=\frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$$

$$=\frac{\frac{1}{3}\times\frac{2}{9}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{2}{9}+\frac{1}{3}\times\frac{1}{6}}=\frac{\frac{2}{9}}{\frac{1}{5}+\frac{2}{9}+\frac{1}{6}}=\frac{20}{53}$$

18. Let the dealer purchases x fans and y sewing machines. Then, the profit function Z is given by

$Z = 22x + 18y$



The two variables x and y satisfy the following constraints: $360x + 240y \le 5760$ or $3x + 2y \le 48$, $x + y \le 20$ and $x \ge 0$, $y \ge 0$

Hence mathematical formulation of the given L.P.P. is:

Maximize $Z = 22x + 18y$, subject to constraints : $3x + 2y \le 48$; $x + y \le 20$ and $x \ge 0$, $y \ge 0$

Feasible region ($OAPD$) is shown shaded in the given figure.

Vertices of the feasible region are $O(0, 0)$, $A(16, 0)$, $P(8, 12)$ and $D(0, 20)$.

$\therefore$ At $O(0, 0)$, $Z = 0$

At $A(16, 0)$, $Z = 22 \times 16 + 18 \times 0 = 352$

At $P(8, 12)$, $Z = 22 \times 8 + 18 \times 12 = 392$

At $D(0, 20)$, $Z = 22 \times 0 + 18 \times 20 = 360$

Thus profit is maximum at $P(8, 12)$.

Hence profit is maximum when 8 fans and 12 sewing machines should be purchased and sold and the maximum profit is ₹ 392.

19. A coin picked up at random from the bag may be a fair coin or a coin having head on both sides. The tossed coin may result in a head in the following mutually exclusive ways.

(i) A fair coin is taken out of the bag and then it is tossed to get a head.

(ii) A two headed coin is taken out of the bag and then it is tossed to get a head.

So, let us define the following events.

E_1 = Taking out a fair coin from the bag.

E_2 = Taking out a coin having head on both sides

A = Tossed coin results in a head.

Clearly, $P(E_1) = \dfrac{n+1}{2n+1}$ and $P(E_2) = \dfrac{n}{2n+1}$.

Also, $P(A/E_1) = 1/2$ and $P(A/E_2) = 1$

By total probability theorem,

$$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$$

$$= \frac{n+1}{2n+1}\times\frac{1}{2}+\frac{n}{2n+1}\times 1=\frac{3n+1}{2(2n+1)}$$

But, it is given that $P(A) = 31/42$.

$$\therefore\quad \frac{31}{42}=\frac{3n+1}{2(2n+1)} \Rightarrow 62n + 31 = 63n + 21 \Rightarrow n = 10.$$

20. It is given that $P(4) = 1/10$ and other scores being equally likely. So, let

$$P(1) = P(2) = P(3) = P(5) = P(6) = p.$$

$$\because\quad P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$

$$\Rightarrow 5p + \frac{1}{10} = 1 \Rightarrow p = \frac{9}{50}$$

Thus, $P(1) = P(2) = P(3) = P(5) = P(6) = \dfrac{9}{50}$

When die is tossed twice, there may be no four or one of the two throws may result in a four or both the throws produce 4. So, X can take values 0, 1 and 2.

Now,

$P(X = 0)$ = Probability of not getting a four in both the throws $= \left(1-\dfrac{1}{10}\right)\left(1-\dfrac{1}{10}\right)=\dfrac{9}{10}\times\dfrac{9}{10}=\dfrac{81}{100}$

$P(X = 1)$ = Probability of getting a four in one of the two throws

$$=\frac{1}{10}\times\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10}\right)\times\frac{1}{10}=\frac{1}{10}\times\frac{9}{10}+\frac{9}{10}\times\frac{1}{10}=\frac{18}{100}$$

$P(X = 2)$ = Probability of getting a four in both the throws $= \dfrac{1}{10}\times\dfrac{1}{10}=\dfrac{1}{100}$

Thus, the probability distribution of X is as given below

X	0	1	2
$P(X)$	81/100	18/100	1/100

Calculation of variance

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	81/100	0	0
1	18/100	18/100	18/100
2	1/100	2/100	4/100
		$\Sigma p_i x_i = \dfrac{20}{100}$	$\Sigma p_i x_i^2 = \dfrac{22}{100}$

Thus, $\Sigma p_i x_i = \dfrac{20}{100} = \dfrac{1}{5}$ and $\Sigma p_i x_i^2 = \dfrac{22}{100} = \dfrac{11}{50}$

$$\therefore\quad \text{Var}(X) = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{11}{50}-\left(\frac{1}{5}\right)^2=\frac{9}{50}=0.18$$

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MPP-9 MONTHLY Practice Problems

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.
Self check table given at the end will help you to check your readiness.

Vector Algebra, Three Dimensional Geometry

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The direction ratios of a normal to the plane through (1, 0, 0), (0, 1, 0), which makes an angle of $\pi/4$ with the plane $x + y = 3$ are
   (a) $<1, \sqrt{2}, 1>$ (b) $<1, 1, \sqrt{2}>$
   (c) $<1, 1, 2>$ (d) $<\sqrt{2}, 1, 1>$

2. If $\vec{b}$ and $\vec{c}$ are unit vectors, then for any arbitrary vector $\vec{a}$, $(((\vec{a}\times\vec{b})+(\vec{a}\times\vec{c}))\times(\vec{b}\times\vec{c}))\cdot(\vec{b}-\vec{c})$ is always equal to
   (a) $|\vec{a}|$ (b) $\frac{1}{2}|\vec{a}|$ (c) $\frac{1}{3}|\vec{a}|$ (d) zero

3. A line makes angles α, β, γ with the coordinate axes. If $\alpha + \beta = \pi/2$, then $(\cos\alpha + \cos\beta + \cos\gamma)^2$ is equal to
   (a) $1 + \sin 2\alpha$ (b) $1 + \cos 2\alpha$
   (c) $1 - \sin 2\alpha$ (d) none of these

4. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three zon-zero vectors such that $|\vec{c}| = 1$, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 = \lambda(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2),$$

   where λ is equal to
   (a) 1/2 (b) 1/4 (c) 1 (d) 2

5. $\vec{a}$ and $\vec{b}$ are two non-collinear unit vectors. If $\vec{\mu}_1 = \vec{a} - (\vec{a}\cdot\vec{b})\vec{b}$ and $\vec{\mu}_2 = \vec{a}\times\vec{b}$, then $|\vec{\mu}_2|$ is always equal to
   (a) $|\vec{\mu}_1| + |\vec{\mu}_1\cdot\vec{a}|$ (b) $|\vec{\mu}_1| + |\vec{\mu}_1\cdot\vec{b}|$
   (c) $|\vec{\mu}_1\cdot(\vec{a}+\vec{b})|$ (d) $|\vec{\mu}_1\cdot(\vec{a}-\vec{b})|$

6. The length of projection of the line segment joining the points (1, 0, –1) and (–1, 2, 2) on the plane $x + 3y - 5z = 6$ is equal to
   (a) 2 (b) $\sqrt{\frac{271}{53}}$ (c) $\sqrt{\frac{472}{31}}$ (d) $\sqrt{\frac{474}{35}}$

One or More Than One Option(s) Correct Type

7. The vectors $(x, x + 1, x + 2)$, $(x + 3, x + 4, x + 5)$ and $(x + 6, x + 7, x + 8)$ are coplanar for
   (a) all values of x (b) $x < 0$
   (c) $x > 0$ (d) none of these

8. Consider the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z = 3$. The plane $67x - 162y + 47z + 44 = 0$ bisects the angle between the given planes which
   (a) contains origin (b) is acute
   (c) is obtuse (d) none of these

9. The line $\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Then which of the following is/are the side of the triangle?
   (a) $\frac{x-7}{2} = \frac{y-2}{-3} = \frac{z-4}{6}$ (b) $\frac{x-7}{3} = \frac{y-2}{6} = \frac{z-4}{2}$
   (c) $\frac{x-7}{3} = \frac{y-2}{5} = \frac{z-4}{-1}$ (d) none of these

10. $\vec{a}, \vec{b}$ and $\vec{c}$ are unimodular and coplanar. A unit vector $\vec{d}$ is perpendicular to them. If $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$, and the angle between $\vec{a}$ and $\vec{b}$ is 30° then $\vec{c}$ =
    (a) $(\hat{i} - 2\hat{j} + 2\hat{k})/3$ (b) $(-\hat{i} + 2\hat{j} - 2\hat{k})/3$
    (c) $(2\hat{i} + 2\hat{j} - \hat{k})/3$ (d) $(-2\hat{i} - 2\hat{j} + \hat{k})/3$

11. If the planes $r\cdot(\hat{i}+\hat{j}+\hat{k})=q_1$, $\vec{r}\cdot(\hat{i}+2a\hat{j}+\hat{k})=q_2$ and $\vec{r}\cdot(a\hat{i}+a^2\hat{j}+\hat{k})=q_3$ intersect in a line, then the value of a is
    (a) 1 (b) 1/2 (c) 2 (d) 0

12. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection at $\vec{a}$ is of magnitude $\sqrt{(2/3)}$ is
    (a) $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$
    (c) $-2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

13. A particle in equilibrium is subjected to four forces
$F_1=10\hat{k}$, $F_2=u\left(\frac{4}{13}\hat{i}-\frac{12}{13}\hat{j}+\frac{3}{13}\hat{k}\right)$,
$F_3=v\left(-\frac{4}{13}\hat{i}-\frac{12}{13}\hat{j}+\frac{3}{13}\hat{k}\right)$ and
$F_4=w(\cos\theta\,\hat{i}+\sin\theta\,\hat{j})$. Then
    (a) $u+v=\frac{130}{3}$, $w=40\operatorname{cosec}\theta$
    (b) $u=\frac{1}{3}-\cot\theta$, $v=\frac{1}{3}+\cot\theta$, $w=40\operatorname{cosec}\theta$
    (c) $u=\frac{1}{3}+\cot\theta$, $v=\frac{1}{3}-\cot\theta$, $w=40\operatorname{cosec}\theta$
    (d) None of these

Comprehension Type

A ray of light comes along the line $L=0$ and strikes the plane mirror kept along the plane $P=0$ at B. $A(2, 1, 6)$ is a point on the line $L=0$ whose image about $P=0$ is A'. It is given that $L=0$ is $\frac{x-2}{3}=\frac{y-1}{4}=\frac{z-6}{5}$ and $P=0$ is $x+y-2z=3$.

14. The coordinates of A' are
    (a) (6, 5, 2) (b) (6, 5, –2)
    (c) (6, –5, 2) (d) none of these

15. The coordinates of B are
    (a) (5, 10, 6) (b) (10, 15, 11)
    (c) (–10, –15, –14) (d) none of these

Matrix Match Type

16. Match the following columns:

	Column-I		Column-II
P.	If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert$ such that angle between each pair of vectors is $\pi/3$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=\sqrt{6}$, then $2\lvert\vec{a}\rvert=$	1.	3
Q.	If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$, $\lvert\vec{a}\rvert=2$, $\lvert\vec{b}\rvert=3$ and $\lvert\vec{c}\rvert=6$, then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert-2=$	2.	2
R.	If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}-4\hat{k}$, $\vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}+2\hat{j}+\hat{k}$, then $\frac{1}{7}(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=$	3.	4
S.	If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert=2$ and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=2$, then $[\vec{a}\ \vec{b}\ \vec{c}]\cos 45°$ is equal to	4.	5

	P	Q	R	S		P	Q	R	S
(a)	2	3	4	1	(b)	3	1	2	4
(c)	2	4	1	3	(d)	3	1	4	2

Numerical Answer Type

17. If $\vec{d}=\lambda(\vec{a}\times\vec{b})+\mu(\vec{b}\times\vec{c})+\nu(\vec{c}\times\vec{a})$, $[\vec{a}\vec{b}\vec{c}]=1/8$ and $\vec{d}\cdot(\vec{a}+\vec{b}+\vec{c})=8$, then $\lambda+\mu+\nu$ is equal to

18. The distance of the point $P(-2, 3, -4)$ from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$ is d, then the value of $(2d-8)$ is

19. If $\vec{a}$ and $\vec{b}$ are vectors perpendicular to each other such that $\lvert\vec{a}\rvert=2$, $\lvert\vec{b}\rvert=3$ and $\vec{c}\times\vec{a}=\vec{b}$, then the least value of $2\lvert\vec{c}-\vec{a}\rvert$ is

20. If the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles then the value of $[p]$, where $[\cdot]$ denotes the greatest integer function, is

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SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

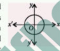
TARGET JEE

Circle

DEFINITION

A circle is defined as the locus of a point which moves in such a way that its distance from a fixed point is always same (constant). The fixed point is called the centre of circle and the constant *i.e.,* fixed distance is known as radius of the circle.

STANDARD EQUATION OF A CIRCLE

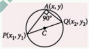


- An equation of circle is said to be in its standard form if its centre lies at origin. The equation of circle centred at origin and having radius 'r' is given by $x^2 + y^2 = r^2$.
- A circle centred at (h, k) and having radius 'r' has the equation $(x - h)^2 + (y - k)^2 = r^2$.

GENERAL EQUATION OF A CIRCLE

- General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, whose centre is $(-g, -f)$ and radius, $r = \sqrt{g^2 + f^2 - c}$.
- Sometime general equation of circle is given by $ax^2 + ay^2 + 2gx + 2fy + c = 0$, whose centre is $\left(-\frac{g}{a}, -\frac{f}{a}\right)$ and radius, $r = \frac{\sqrt{g^2 + f^2 - ac}}{a}$.
  - If $g^2 + f^2 - ac > 0$, circle is real.
  - If $g^2 + f^2 - ac = 0$, circle is point circle.
  - If $g^2 + f^2 - ac < 0$, circle is imaginary.

Diametric Form of Equation of Circle



Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be extremities of diameter of circle and $A(x, y)$ be any point on the circle then equation of circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$, whose centre is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

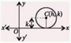
and radius is

$$\frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{2}.$$

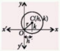
VARIOUS FORMS OF EQUATION OF CIRCLE

The equation of circle centred at (h, k) and radius r is

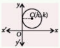
$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{...(i)}$$



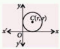
- When circle touches x-axis : In this case $r = k$ and (i) reduces to $(x - h)^2 + (y - r)^2 = r^2$
  or $x^2 + y^2 - 2xh - 2yr + h^2 = 0$



- When circle passes through origin : Here $h^2 + k^2 = r^2$ and (i) reduces to $(x - h)^2 + (y - k)^2 = h^2 + k^2$
  or $x^2 + y^2 - 2hx - 2yk = 0$



- When circle touches y-axis : In this case $h = r$ and (i) reduces to $(x - r)^2 + (y - k)^2 = r^2$
  or $x^2 + y^2 - 2rx - 2ky + k^2 = 0$



- When circle touches both axes : In this case $h = k = r$ and (i) reduces to $(x - r)^2 + (y - r)^2 = r^2$
  or $x^2 + y^2 - 2r(x + y) + r^2 = 0$

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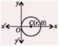
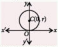
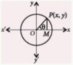
- When circle passes through origin and centred at x-axis : Here $k = 0$, $h = r$ and (i) reduces to $x^2 + y^2 - 2rx = 0$



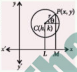
- When circle passes through origin and centred at y-axis : Here $h = 0$, $k = r$ and (i) reduces to $x^2 + y^2 - 2ry = 0$



- Parametric form of equation of circle centred at origin : The equation $x = r\cos\theta$, $y = r\sin\theta$ are called parametric equations of circle $x^2 + y^2 = r^2$, where θ is called parameter and $(x, y) = (r\cos\theta, r\sin\theta)$ is known as parametric co-ordinates and $0 \le \theta < 2\pi$.



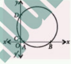
- Parametric form of equation of circle centred at $C(h, k)$ and radius 'r' : The parametric co-ordinates of any point on the circle $(x - h)^2 + (y - k)^2 = r^2$ are $x = h + r\cos\theta$, $y = k + r\sin\theta$ $(0 \le \theta < 2\pi)$ *i.e.*, $P(x, y) = P(h + r\cos\theta, k + r\sin\theta)$.



INTERCEPTS MADE BY CIRCLE ON THE AXES

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be equation of circle, then

- Intercept made by circle with x-axis is $AB = 2\sqrt{g^2 - c}$
- Intercept made by circle with y-axis is $CD = 2\sqrt{f^2 - c}$



Note :

(i) If $g^2 = c$, then circle touches x-axis.

(ii) If $g^2 > c$, then circle meets x-axis at two real and distinct points.

(iii) If $g^2 < c$, then circle does not meet x-axis at real points and the circle lies either above or below x-axis.

(iv) Case is similar for intercept on y-axis.

(v) If $g^2 = f^2 = c$, then circle touches both axes and intercept made by the circle on both axes is zero.

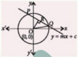
INTERCEPT MADE BY A LINE WITH THE CIRCLE

Let $y = mx + c$ be the line and $x^2 + y^2 = r^2$ be equation of circle, PQ intercept made by line, then

$$PQ = 2PR = 2QR$$

$$= 2\sqrt{OQ^2 - OR^2}$$

$$= 2\sqrt{\frac{r^2(1+m^2) - c^2}{1+m^2}}$$



Note : If $PQ = 0$, then line touches the circle *i.e.*, line will be tangent to the circle and condition of tangency can be deduce by keeping

$$2\sqrt{\frac{r^2(1+m^2) - c^2}{1+m^2}} = 0 \Rightarrow c^2 = r^2(1+m^2)$$

and the point of tangency is $\left(-\frac{mr^2}{c}, \frac{r^2}{c}\right)$.

POSITION OF A POINT WITH RESPECT TO A CIRCLE

Let $P(x_1, y_1)$ be any point, $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be equation of a circle and r be radius, then

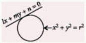
- The point $P(x_1, y_1)$ lies on the circle if $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$

- The point $P(x_1, y_1)$ lies outside the circle if $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c > 0$

- The point $P(x_1, y_1)$ lies inside the circle if $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$

LINE AND CIRCLE

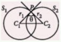
- Let $lx + my + n = 0$ be a line and $x^2 + y^2 = r^2$ be the equation of circle, then line will touch the circle if $n^2 = r^2(l^2 + m^2)$.



- The line $lx + my + n = 0$ will touch the circle $ax^2 + ay^2 + 2gx + 2fy + c = 0$ if $(lg + mf - an)^2 = (l^2 + m^2)(g^2 + f^2 - a^2c)$.
  If $a = 1$ then condition reduces to $(lg + mf - n)^2 = (l^2 + m^2)(g^2 + f^2 - c)$.

ANGLE OF INTERSECTION OF TWO CIRCLES (CURVES)

If two curves S_1 and S_2 intersect at a point P and PC_1, PC_2 are the tangents at P, then the angle between the tangents at P is called angle of intersection of the two curves. If θ be the angle of intersection, then



$$\cos\theta = \frac{r_1^2 + r_2^2 - (C_1C_2)^2}{2r_1r_2}.$$

Condition for two circles intersect at two points :

A necessary and sufficient condition for the two circles to intersect at two distinct points is

$$r_1 + r_2 > C_1C_2 > |r_1 - r_2|$$

where C_1, C_2 are centres and r_1, r_2 are radii of circles.

Condition for two intersecting circles to be orthogonal :

Let $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and
$S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ be two circles.

Centre of $S_1 = C_1(-g_1, -f_1)$ and

$$r_1 = \sqrt{g_1^2 + f_1^2 - c_1} \Rightarrow r_1^2 = g_1^2 + f_1^2 - c_1$$

Centre of $S_2 = C_2(-g_2, -f_2)$ and

$$r_2 = \sqrt{g_2^2 + f_2^2 - c_2} \Rightarrow r_2^2 = g_2^2 + f_2^2 - c_2$$

Circles $S_1 = 0$ and $S_2 = 0$ be orthogonal if tangents at their point of intersection (made) form 90° angle between them.

Now, $r_1^2 + r_2^2 = (C_1C_2)^2$

$$\Rightarrow r_1^2 + r_2^2 = (g_1 - g_2)^2 + (f_1 - f_2)^2$$

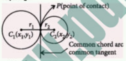
$$\Rightarrow r_1^2 + r_2^2 = g_1^2 + f_1^2 + g_2^2 + f_2^2 - 2g_1g_2 - 2f_1f_2$$

$$\Rightarrow r_1^2 + r_2^2 = (g_1^2 + f_1^2 - c_1) + (g_2^2 + f_2^2 - c_2) + c_1 + c_2 - 2g_1g_2 - 2f_1f_2$$

$\Rightarrow 2g_1g_2 + 2f_1f_2 = c_1 + c_2$, which is the required condition of orthogonality of two circles.

EXTERNAL AND INTERNAL POINT OF CONTACT OF CIRCLES

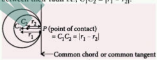
- The point of contact of two circles touching each other externally. Let $C_1(x_1, y_1)$, $C_2(x_2, y_2)$ are centres of circles with radii r_1 and r_2 respectively, then they touch each other externally if distance between their centres is equal to the sum of their radii *i.e.*, $C_1C_2 = r_1 + r_2$.



  Their point of contact is

$$P \equiv \left(\frac{r_1x_2 + r_2x_1}{r_1 + r_2}, \frac{r_1y_2 + r_2y_1}{r_1 + r_2}\right)$$

- The circles touch each other internally if distance between their centre is equal to the difference between their radii *i.e.*, $C_1C_2 = |r_1 - r_2|$.



  Their point of contact is $P \equiv \left(\frac{r_1x_2 - r_2x_1}{r_1 - r_2}, \frac{r_1y_2 - r_2y_1}{r_1 - r_2}\right)$

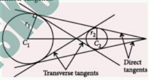
COMMON TANGENTS TO THE CIRCLES

Let S_1 and S_2 be two circles whose centres are C_1 and C_2 and radii are r_1 and r_2 respectively. These circles having three types of tangents.

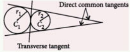
- **Direct common tangent** is a tangent touching two circles and not intersecting the line of centres between them.
- **Transverse common tangent** is a tangent touching two circles and intersect the line of centres.
- **Common tangent** is a tangent touching two circles at a same point and such type of tangent exist only when circles touches each other either externally or internally.

Now the following cases appears :

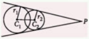
- If $C_1C_2 > r_1 + r_2$ and $r_1 \neq r_2$ or $r_1 = r_2$. In this case total common tangents are 4 out of which 2 are direct common tangents and 2 are transverse common tangents.



- If $C_1C_2 = r_1 + r_2$, total number of common tangents are 3 out of which 2 are direct common tangents and 1 transverse common tangent. In such case circle touches each other externally.



- If $|r_1 - r_2| < C_1C_2 < r_1 + r_2$, total number of common tangents are 2 which are direct tangents.



  In this case circles intersect each other at two distinct points.

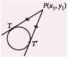
- If $|r_1 - r_2| = C_1C_2$, in this case there is only 1 common tangent which is real tangent. In this case circles touch each other internally.

- If $C_1C_2 < |r_1 - r_2|$ in this case one circle lies inside the other and no common tangent exist.

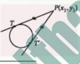
EQUATION OF TANGENTS AND NORMALS

- Equation of tangent to the circle $x^2 + y^2 = r^2$ at a point $P(x_1, y_1)$ is given by $xx_1 + yy_1 = r^2$ which is also known as chord of contact of tangent.
- Equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at a point $P(x_1, y_1)$ is given by $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.
- Equation of normal to the circle $x^2 + y^2 = r^2$ at a point $P(x_1, y_1)$ is $xy_1 - yx_1 = 0$ or $\frac{x}{x_1} = \frac{y}{y_1}$.
- Equation of normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at a point $P(x_1, y_1)$ is $\frac{y - y_1}{y_1 + f} = \frac{x - x_1}{x_1 + g}$.

LENGTH OF TANGENT TO THE CIRCLE FROM A GIVEN POINT



- Let $P(x_1, y_1)$ be a point and $x^2 + y^2 = r^2$ be equation of circle, then length of tangent is $PT = PT' = \sqrt{x_1^2 + y_1^2 - r^2}$.

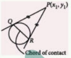


- Let equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ and $P(x_1, y_1)$ be any point from which tangents are drawn to the circle, then length of tangent is $PT = PT' = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$.

FAMILY OF CIRCLES UNDER DIFFERENT CONDITIONS

- Let $S_1 = 0 = S_2$ are two given circles then family of circles passes through the intersection of these two circles is given by $S_1 + \lambda S_2 = 0$ or $S_2 + \lambda S_1 = 0$, where λ is a parameter, $(\lambda \neq -1)$
- Equation of family of circles passing through $S = 0$ and a line $L = 0$ is given by $S + \lambda L = 0$.
- Equation of family of circles passing through two given points (x_1, y_1), (x_2, y_2) is given by $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda L = 0$, where L is the equation of line passing through (x_1, y_1), (x_2, y_2) and λ is a parameter.
- Equation of family of circles touches the line $lx + my + n = 0$ at a point (x_1, y_1) is given by $(x - x_1)^2 + (y - y_1)^2 + \lambda(lx + my + n) = 0$.

PAIR OF TANGENTS TO THE CIRCLE (COMBINED EQUATION OF TANGENTS)



- Let $P(x_1, y_1)$ be a point outside the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ then, their combined equation is given by $SS_1 = T^2$, where
  $S \equiv x^2 + y^2 + 2gx + 2fy + c$,
  $T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$
  and $S_1 \equiv {x_1}^2 + {y_1}^2 + 2gx_1 + 2fy_1 + c$
- These tangents will be perpendicular if coefficient of x^2 + coefficient of $y^2 = 0$.

DIRECTOR CIRCLE

- The locus of the point of intersection of two perpendicular tangents to a circle is called the director circle.

  If equation of circle be $x^2 + y^2 = r^2$, then equation of director circle is $x^2 + y^2 = 2r^2$.

Note :

(i) Director circle is concentric circle whose radius is $\sqrt{2}$ times the radius of the given circle.

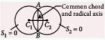
(ii) The equation of director circle of $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by
$x^2 + y^2 + 2gx + 2fy + 2c - g^2 - f^2 = 0$

RADICAL AXIS

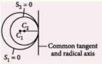
- Radical axis of two circles is the locus of a point which moves such that lengths of tangents drawn from it to the two circles are equal.

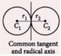
  Let the equation of two circles be
  $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and
  $S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

  If $P(x_1, y_1)$ is the point on the radical axis of these two circles, then
  $$\sqrt{x_1^2 + y_1^2 + 2g_1x_1 + 2f_1y_1 + c_1} = \sqrt{x_1^2 + y_1^2 + 2g_2x_1 + 2f_2y_1 + c_2}$$
  On squaring both sides and simplifying, we get
  $$2(g_1 - g_2)x_1 + 2(f_1 - f_2)y_1 + c_1 - c_2 = 0 \qquad \text{...(*)}$$
  $\therefore$ The locus of $P(x_1, y_1)$ *i.e.*, the equation of the radical axis of two circles is
  $2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$,
  which is the equation obtained from $S_1 - S_2 = 0$.
- If circles $S_1 = 0$ and $S_2 = 0$ intersect in real and distinct points then $S_1 - S_2 = 0$ is the equation of common chord of two circles.

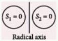


- If two circles touches each other internally then common tangent is their radical axis.





- If two circles do not intersect then $S_1 - S_2 = 0$ is only radical axis.



- Radical axis of two circles is always perpendicular to the line joining their centres.

  Slope of radical axis $= -\dfrac{(g_1 - g_2)}{f_1 - f_2}$ (say m_1) and slope of line joining $C_1(-g_1, -f_1)$,

  $$C_2(-g_2, -f_2) = \frac{f_1 - f_2}{g_1 - g_2} \text{ (say } m_2\text{)}$$

  $\therefore\ m_1 m_2 = -1$ *i.e.* $(S_1 - S_2) \perp C_1C_2$
- Radical axis bisect a common tangent between the two circles.
- Radical axis will pass through the mid point of the line joining the centre of two circles only if circles have equal radii and touches each other externally.
- If two circles $S_1 = 0$ and $S_2 = 0$ are concentric then they have no radical axis.
- Let $S_1 = 0$, $S_2 = 0$, $S_3 = 0$ be three circles then their radical axis taken in pair are either parallel or concurrent. Radical axis are parallel iff the centres of the circles are collinear.

Radical Centre

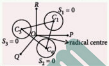
The common point of intersection of the radical axis of three circles taken two at a time is called radical centre of three circles.

Let $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$
$S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$
$S_3 = x^2 + y^2 + 2g_3x + 2f_3y + c_3 = 0$
be three equations of circles.

$\therefore$ Radical axis of $S_1 = 0$ and $S_2 = 0$ is $S_1 - S_2 = 0$
i.e., $2(g_1 - g_2) + 2(f_1 - f_2) + c_1 - c_2 = 0$
Radical axis of $S_2 = 0$ and $S_3 = 0$ is $S_2 - S_3 = 0$
i.e., $2(g_2 - g_3) + 2(f_2 - f_3) + c_2 - c_3 = 0$
and radical axis of $S_1 = 0$ and $S_3 = 0$ is $S_1 - S_3 = 0$
i.e., $2(g_1 - g_3) + 2(f_1 - f_3) + c_1 - c_3 = 0$

Now $S_1 - S_2 = 0$, $S_2 - S_3 = 0$ are straight lines and their point of intersection must satisfy the equation $(S_1 - S_2) + (S_2 - S_3) = 0$.

Thus, point of intersection satisfies the equation $S_1 - S_3 = 0$, which is the radical axis of $S_1 = 0$ and $S_3 = 0$. Thus radical axis of three circles are concurrent and their point of concurrency is the radical centre.



The lines OP, OQ, OR passes through O which is the radical centre.

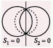
CO-AXIAL SYSTEM OF CIRCLES

A system of circles $S_1 = 0, S_2 = 0, \ldots, S_n = 0$ is said to be family of co-axial circles if each two of them have same radical axis.

- Equation of system of co-axial circle having equation of radical axis $ax + by + c = 0$ and a circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$ is given by

  $$S + \lambda(ax + by + c) = 0,$$

  where λ is a parameter or an arbitrary constant.



- Equation of family of co-axial circles when equation of two circles is given is formed as (equation of any one of the given circle) + λ (equation of radical axis of two circles) = 0

  Let $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and
  $S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$,
  then equation of co-axial circle is
  $S_1 + \lambda(S_2 - S_1) = 0$ or $S_2 + \lambda(S_1 - S_2) = 0$

Limiting Points

- The coordinates of the centre of co-axial circles is called the limiting point. For limiting points radius of co-axial system of circles vanished, which means that limiting points of a co-axial system are the centres of point circles.
- The limiting points of the co-axial system of circles $x^2 + y^2 + 2gx + 2fy + c = 0$ are given by $\left(-\dfrac{gc}{g^2 + f^2}, -\dfrac{fc}{g^2 + f^2}\right)$.

SOME MORE RESULTS ABOUT CIRCLES

- Length of an external common tangent to the two circles whose radii are r_1 and r_2 and distance between centres is d is $\sqrt{d^2 - (r_1 - r_2)^2}$.
- Length of an internal common tangent of two circles whose radii are r_1 and r_2 and distance between their centres is d is $\sqrt{d^2 - (r_1 + r_2)^2}$.

- If two tangents drawn from origin (0, 0) to the circle $x^2+y^2+2gx+2fy+c=0$ are perpendicular to each other, then $g^2+f^2=2c$.
- If tangent to the circle $x^2+y^2=r^2$ at a point (α, β) meet the co-ordinate axes the points P and Q and O be the origin, then area of the ΔOPQ equals $\frac{r^4}{2\alpha\beta}$.
- The angle between the tangents from a point (a, b) to the circle $x^2+y^2=r^2$ is given by $2\tan^{-1}\left(\frac{r}{\sqrt{a^2+b^2-r^2}}\right)$.
- If OP and OQ are tangents from (0, 0) to the circle $x^2+y^2+2gx+2fy+c=0$ and R be the centre of the circle, then area of quadrilateral $OPRQ$ is $\sqrt{c(g^2+f^2-c)}$.
- If circles $x^2+y^2+2gx+k^2=0$ and $x^2+y^2+2fy+k^2=0$ touches each other, then $\frac{1}{g^2}+\frac{1}{f^2}=\frac{1}{k^2}$.
- If the line $lx+my+n=0$ is tangent to the circle $(x-h)^2+(y-k)^2=r^2$ then $(hl+km+n)^2=r^2(l^2+m^2)$.
- If tangents from origin to the circle $x^2+y^2+2gx+2fy+c=0$ meet at points A and B, then circumcentre of ΔOAB is $\left(\frac{-g}{2}, \frac{-f}{2}\right)$.
- The length of the chord intercepted by the circle $x^2+y^2=r^2$ on the line $\frac{x}{a}+\frac{y}{b}=1$ is $2\times\frac{\sqrt{r^2(a^2+b^2)-a^2b^2}}{a^2+b^2}$.
- The length of common chord of circles $(x-\alpha)^2+y^2=\alpha^2$ and $x^2+(y-\beta)^2=\beta^2$ is equal to $\frac{2\alpha\beta}{\sqrt{\alpha^2+\beta^2}}$.

PROBLEMS

Single Correct Answer Type

1. If A and B are the points of intersection of circles $x^2+y^2+3x+7y+2P-5=0$ and $x^2+y^2+2x+2y-P^2=0$, then there is a circle passing through A, B and (1, 1) for

(a) all except one value of P

(b) all except two values of P

(c) all values of P

(d) exactly one value of P

2. A circle passes through $A(1, 0)$, $B(5, 0)$ and touches y-axis at $(0, h)$, then

(a) $h=\sqrt{3}$

(b) $h=\sqrt{5}$

(c) $h=\sqrt{10}$

(d) None of these

3. If the radical axis of the circles $x^2+y^2+2gx+2fy+c=0$ and $2x^2+2y^2+3x+8y+2c=0$ touches the circle $(x+1)^2+(y+1)^2=1$, then ordered pair (f, g) equals to

(a) $\left(2, \frac{3}{4}\right)$

(b) $\left(-2, \frac{-3}{4}\right)$

(c) $\left(\frac{3}{4}, 2\right)$

(d) $\left(\frac{-3}{4}, -2\right)$

4. The equation of circle belonging to the family of co-axial circle if the co-ordinates of the limiting points of co-axial is (1, –2) and (2, 0) which passes through origin is

(a) $x^2+y^2-12x-16y=0$

(b) $x^2+y^2-12x+16y=0$

(c) $x^2+y^2+12x+16y=0$

(d) None of these

More Than One Correct Answer Type

5. If the circles $S=x^2+y^2+2gx+2fy+C=0$ cuts each of the circles $S_1=x^2+y^2-8=0$, $S_2=x^2+y^2-4x-6y+12=0$ and $S_3=x^2+y^2+2x-4y-2=0$ at the end points of a diameter, then

(a) $C=-8$

(b) $g+f=C+7$

(c) $g^2+f^2-C=9$

(d) $g-f=1$

Comprehension Type

Paragraph for Q. No. 6 to 10

If $S_1=0$ and $S_2=0$ are equations of circles intersecting in real and distinct points A and B then AB is common chord of the circles $S_1=0$, $S_2=0$ known as radical axis of circles which is ⊥ to line segment joining the centres of the circles. If $S_1=x^2+y^2+2gx+2fy+c=0$ and $S_2=x^2+y^2+2g'x+2f'y+c'=0$ then equation of radical axis is given by $2x(g-g')+2y(f-f')+c-c'=0$. If circles touching each other externally or internally then point of contact can be determined by section formula. Consider the circles $S_1=x^2+y^2-4x-2y+4=0$ and $S_2=x^2+y^2-12x-8y+36=0$ on the basis of above informations answer the following questions.

6. The point at which the circles $S_1=0$, $S_2=0$ touches each other is

(a) $\left(\frac{8}{5}, \frac{14}{5}\right)$

(b) $\left(\frac{14}{5}, \frac{8}{5}\right)$

(c) $\left(-\frac{8}{5}, \frac{14}{5}\right)$

(d) $\left(\frac{8}{5}, -\frac{14}{5}\right)$

MPP-9 CLASS XII ANSWER KEY

1. (b) 2. (d) 3. (a) 4. (a) 5. (b)
6. (d) 7. (a,b,c) 8. (a,b) 9. (a,b) 10. (a, b)
11. (a,b) 12. (a,c) 13. (a) 14. (b) 15. (c)
16. (c) 17. (64) 18. (9) 19. (3) 20. (6)

7. If the direct common tangents to $S_1 = 0$ and $S_2 = 0$ intersect at $A(x, y)$, then the coordinate of A is

(a) $\left(0, \frac{4}{3}\right)$ (b) $\left(0, -\frac{2}{3}\right)$ (c) $\left(\frac{2}{3}, 0\right)$ (d) $\left(\frac{4}{3}, 0\right)$

8. The equation of direct common tangent to the circles $S_1 = 0$ and $S_2 = 0$ is

(a) $y = 2$ (b) $x = 0$
(c) $24x - 7y = 16$ (d) $24x + 7y = 16$

9. The equation of radical axis of the circle is

(a) $4x + 3y - 16 = 0$ (b) $4x - 3y + 16 = 0$
(c) $3x - 4y - 2 = 0$ (d) $4x + 3y + 2 = 0$

10. The number of common tangents that can be drawn to the circles $S_1 = 0 = S_2$ is/are

(a) 1 (b) 2 (c) 3 (d) 4

Paragraph for Q. No. 11 and 12

Let S be the circle passes through $A(-1, 2)$ and $B(2, 3)$. Answer the following question

11. If the radius of the circle S is $\sqrt{5}$, then length of its intercept on the x-axis is

(a) 4 (b) 3 (c) 2 (d) 1

12. If S touches the line $y = -x$, then length of its intercept on x-axis is

(a) 2/3 (b) 4/3 (c) 1/2 (d) 2

Matrix Match Type

13. Match the following :

Column-I		Column-II	
A.	If radical centres of the circles $S_1 = x^2 + y^2 - 7x - 6y - 4 = 0$, $S_2 = x^2 + y^2 - 8x - 4y - 4 = 0$ and $S_3 = x^2 + y^2 + 10x + 6y - 4 = 0$ is (l, m), then	(p)	$l + m = 20$
B.	The minimum and maximum distance from the point (10, 6) to the circle $x^2 + y^2 - 8x + 4y + 4 = 0$ are l and m, then	(q)	$l + m = 0$
C.	If the minimum and maximum distance from the point (2, –6) to the circle $x^2 + y^2 - 14x - 12y - 540 = 0$ are l and m, then	(r)	$m - l = 8$
		(s)	$l + m = 50$
		(t)	$m - l = 26$

Numerical Answer Type

14. If the equation of the circle whose radius is 4 and which touches the circle $x^2 + y^2 - 6x + 8y + 9 = 0$ at the point (3, 0) is $(x - h)^2 + (y - k)^2 = 16$, then value of $h^2 + k^2$ equals to

15. The length of intercept on the straight line $5x - 3y + 2 = 0$ by the circle $x^2 + y^2 - 4x - 8y + 11 = 0$ is $3k$, then k equals to

SOLUTIONS

1. (a) : Equation of radical axis of the circles $S_1 = x^2 + y^2 + 3x + 7y + 2P - 5 = 0$ and $S_2 = x^2 + y^2 + 2x + 2y - P^2 = 0$ is given by $S_1 - S_2 = 0$

$\Rightarrow\ x + 5y + 2P + P^2 - 5 = 0$ ...(i)

If there is a circle passes through A, B and (1, 1) it is necessary and sufficient that (1, 1) does not lie on AB (*i.e.* $S_1 - S_2 = 0$)

$\therefore\ 1 + 5(1) + 2P + P^2 - 5 \neq 0$

$\Rightarrow\ P^2 + 2P + 1 \neq 0 \Rightarrow P \neq -1$, thus for all values of P except –1, there is a circle passing through A and B and (1, 1).

2. (b) : Equation of line through $A(1, 0)$ and $B(5, 0)$ is $y = 0$ ($\because\ m = 0$ and using $y - 0 = m(x - 1)$)

$\therefore$ Equation of any circle passing through $A(1, 0)$, $B(5, 0)$ is $(x - 1)(x - 5) + (y - 0)(y - 0) + \lambda y = 0$

i.e., $x^2 + y^2 - 6x + \lambda y + 5 = 0$.

This circle will touch y-axis at $(0, h)$ if $h^2 + \lambda h + 5 = 0$ have equal roots (Discriminant = 0)

$\therefore\ \lambda^2 - 20 = 0 \Rightarrow \lambda^2 = 20 \Rightarrow \lambda = \pm 2\sqrt{5}$

$\therefore\ h^2 \pm 2\sqrt{5}\,h + 5 = 0 \Rightarrow (h \pm \sqrt{5})^2 = 0 \Rightarrow h = \pm\sqrt{5}$

3. (a) : We know that if $S_1 = 0 = S_2$ are equation of two circles, then equation of their radical axis is given by $S_1 - S_2 = 0$.

Here $S_1 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$
and $S_2 \equiv 2x^2 + 2y^2 + 3x + 8y + 2c = 0$
$\Rightarrow\ x^2 + y^2 + (3/2)x + 4y + c = 0$
$\therefore\ S_1 - S_2 = 0 \Rightarrow (4g - 3)x + 4(f - 2)y = 0$ ...(i)

Now it is given that radical axis (i) touches the circle $(x + 1)^2 + (y + 1)^2 = 1$ whose centre is (–1, –1) and radius $r = 1$

$$\therefore\ \frac{(4g-3)(-1) + 4(f-2)(-1)}{\sqrt{(4g-3)^2 + 16(f-2)^2}} = \pm 1 \Rightarrow g = 3/4, f = 2$$

4. (b) : Equation of point circles whose limiting point are (1, –2) and (2, 0) is

$(x - 1)^2 + (y + 2)^2 = 0$ and $(x - 2)^2 + y^2 = 0$

$\therefore$ Equation of co-axial circles is

$(x - 1)^2 + (y + 2)^2 + \lambda[(x - 2)^2 + y^2] = 0$ ...(i)

Now, (i) passes through (0, 0)

$\therefore\ 1 + 4 + \lambda(4) = 0 \Rightarrow \lambda = -5/4$

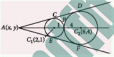
$\therefore$ Required equation of circle is

$$(x-1)^2 + (y+2)^2 - \frac{5}{4}[(x-2)^2 + y^2] = 0$$

$\Rightarrow\ x^2 + y^2 - 12x + 16y = 0.$

5. (a, b, c, d) : Given $S = x^2 + y^2 + 2gx + 2fy + C = 0$,
$S_1 = x^2 + y^2 - 8 = 0 \Rightarrow C_1(0, 0)$
$S_2 = x^2 + y^2 - 4x - 6y + 12 = 0 \Rightarrow C_2(2, 3)$
and $S_3 = x^2 + y^2 + 2x - 4y - 2 = 0 \Rightarrow C_3(-1, 2)$
Now common chords of the circles are
$S - S_1 = 2gx + 2fy + C + 8 = 0$ ...(i)
$S - S_2 = (2g + 4)x + (2f + 6)y + C - 12 = 0$ ...(ii)
$S - S_3 = (2g - 2)x + (2f + 4)y + C + 2 = 0$ ...(iii)
Now for cutting the extremities of the diameter, the chords (i), (ii), (iii) passes through the centres of the circles S_1, S_2 and S_3 respectively.
$\therefore$ $2g(0) + 2f(0) + C + 8 = 0 \Rightarrow C = -8$,
$(2g + 4)(2) + (2f + 6)(3) + C - 12 = 0$
$\Rightarrow 4g + 6f + 6 = 0$ ...(iv)
and $(2g - 2)(-1) + (2f + 4)(2) + C + 2 = 0$
$\Rightarrow -2g + 4f + 4 = 0$ ...(v)
Solving (iv) and (v) we get $f = -1, g = 0$
$\therefore$ $S = x^2 + y^2 + 2gx + 2fy + C = 0$ reduces to $x^2 + y^2 - 2y - 8 = 0$
Now, $C = -8, g = 0, f = -1 \Rightarrow g - f = 1$,
$g + f = -1 = -8 + 7 = C + 7$ and $g^2 + f^2 - C = 9$

6. (b) : Given $S_1 = x^2 + y^2 - 4x - 2y + 4 = 0$
$\Rightarrow C_1(2, 1)$ and $r_1 = 1$
$S_2 = x^2 + y^2 - 12x - 8y + 36 = 0 \Rightarrow C_2(6, 4)$ and $r_2 = 4$
Now $r_1 + r_2 = 1 + 4 = 5 = C_1C_2$ (distance between the centres of circles.



As $r_1 + r_2 = C_1C_2$ which means circles touch each other externally at a point say P which divides C_1C_2 in the ratio 1 : 4
The co-ordinates of P
$$= \left(\frac{1 \times 6 + 4 \times 2}{5}, \frac{1 \times 4 + 4 \times 1}{5}\right) = \left(\frac{14}{5}, \frac{8}{5}\right)$$

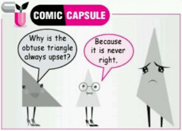
7. (c) : $A(x, y)$ divides C_1C_2 externally in the ratio 1 : 4
$$\therefore A(x, y) = A\left(\frac{1 \times 6 - 4 \times 2}{1 - 4}, \frac{1 \times 4 - 4 \times 1}{1 - 4}\right) = A\left(\frac{2}{3}, 0\right)$$

8. (c) : The direct common tangents meet (intersect) each other at $\left(\frac{2}{3}, 0\right)$, let its slope be m and its equation is given by $y - 0 = m\left(x - \frac{2}{3}\right)$ or $y = mx - \frac{2m}{3}$
As direct common tangents are perpendicular to the lines joining the point of contact and centres of the circles and the distance from point of contact to centres is equal to the radii of circles.
$\therefore$ Distance from (2, 1) to the tangent line $mx - y - \frac{2m}{3} = 0$ is equal to 1.
$$\therefore \frac{2m - 1 - \frac{2m}{3}}{\sqrt{1 + m^2}} = 1 \Rightarrow m = 0, m = \frac{24}{7}$$
$\therefore$ Equations of direct common tangents to S_1 and S_2 are $y = 0$ and $y = \frac{24}{7}x - \frac{16}{7} \Rightarrow 24x - 7y = 16$.

9. (a) : Radical axis of circles $S_1 = 0$ and $S_2 = 0$ is $S_1 - S_2 = 0$
$\therefore$ $-4x + 12x - 2y + 8y + 4 - 36 = 0 \Rightarrow 8x + 6y - 32 = 0$
$\therefore$ Equation of radical axis is $4x + 3y - 16 = 0$.

10. (c) : Circles touches each other externally.
$\therefore$ Total number of common tangents
= 2 direct tangent + 1 transverse tangent
$\therefore$ Total number of common tangents = 3.

11. (a) : Given $A = (-1, 2)$ and $B(2, 3)$
$\therefore$ Equation of AB is $x - 3y + 7 = 0$
$\therefore$ The equation of circle is
$(x + 1)(x - 2) + (y - 2)(y - 3) + \lambda(x - 3y + 7) = 0$
$\Rightarrow S = x^2 + y^2 + (\lambda - 1)x - (3\lambda + 5)y + (7\lambda + 4) = 0$ ...(i)
Now it is given that radius of circle S is $\sqrt{5}$
$\therefore$ $g^2 + f^2 - c = r^2$
$$\Rightarrow \frac{(\lambda - 1)^2 + (3\lambda + 5)^2}{4} - (7\lambda + 4) = 5,$$
where $g = -\left(\frac{\lambda - 1}{2}\right)$ and $f = \frac{3\lambda + 5}{2}$
$\Rightarrow \lambda^2(1 + 9) + \lambda(28) - 4(7\lambda + 4) + 26 = 20$
$\Rightarrow 10\lambda^2 = 10 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = 1, -1$
For $\lambda = 1$, $S = x^2 + y^2 - 8y + 11 = 0$ which does not intersect x-axis.
For $\lambda = -1$, $S = x^2 + y^2 - 2x - 2y - 3 = 0$
$\therefore$ $x^2 - 2x - 3 = 0$ (for intercept on x-axis, $y = 0$)



COMIC CAPSULE
Why is the obtuse triangle always upset?
Because it is never right.

$\Rightarrow \quad x = -1, 3$

$\therefore$ Length of intercept = 3 - (-1) = 4.

12. (c) : $\because \quad S = x^2 + y^2 + (\lambda - 1)x - (3\lambda + 5)y + (7\lambda + 4) = 0$ [(from (i)]

and it touches $y = -x$ ...(ii)

Using (ii) in (i), we get

$2x^2 + 4(\lambda + 1)x + 7\lambda + 4 = 0$ whose roots are equal.

$\therefore D = 0 \Rightarrow 2\lambda^2 - 3\lambda - 2 = 0 \Rightarrow (\lambda - 2)(2\lambda + 1) = 0$

$$\therefore \quad \lambda = 2, -\frac{1}{2}$$

Now for $\lambda = 2$, $y = 0$, from (i) we have $x^2 + x + 18 = 0$ which has no real values. So S does not intersect on x-axis.

Again for $\lambda = -1/2$, $y = 0$ equation (i) reduces to $2x^2 - 3x + 1 = 0 \Rightarrow (x - 1)(2x - 1) = 0 \Rightarrow x = 1, 1/2$

13. A → (q); B → (p, r); C → (s, t)

(A) Radical axis of two circles is a line ⊥ to the line joining the centre of the circle and radical centre of these circles is the point of concurrency of radical axis when two circles taken at a time.

$\therefore$ Radical axis of S_1 and S_3 is $S_1 - S_3 = 0$

$\Rightarrow \quad 17x + 12y = 0.$

Similarly the other radical axis equation are $x - 2y = 0$ and $9x + 5y = 0$. Each radical axis passes through (0, 0)

$\therefore$ Radical centre of the circles S_1, S_2 and S_3 is origin.

$\therefore \quad l = m = 0 \Rightarrow l + m = 0$

(B) Given equation of circle is

$x^2 + y^2 - 8x + 4y + 4 = 0 \Rightarrow (x^2 - 8x) + (y^2 + 4y) + 4 = 0$

$\Rightarrow \quad (x - 4)^2 + (y + 2)^2 = -4 + 16 + 4$

$\Rightarrow \quad (x - 4)^2 + (y + 2)^2 = 16$

$\therefore$ Centre (4, –2) and radius, $r = 4$

Let $S = x^2 + y^2 - 8x + 4y + 4$

Since $(S)_{10,\,6} = 100 + 36 - 80 + 24 + 4 > 0$, therefore the point lies out side the circle.

Now, minimum distance of the point from the circle

$$l = CP - r = \sqrt{(10 - 4)^2 + (6 + 2)^2} - 4 = 10 - 4 = 6$$

and the maximum distance $m = CP + r = 10 + 4 = 14$

$\therefore \quad l + m = 20,\ m - l = 8$

(C) Given equation of circle is

$x^2 + y^2 - 14x - 12y - 540 = 0$

$\Rightarrow \quad (x^2 - 14x) + (y^2 - 12y) = 540$

$\Rightarrow \quad (x - 7)^2 + (y - 6)^2 = 625$

$\Rightarrow \quad (x - 7)^2 + (y - 6)^2 = 25^2$

$\therefore$ Centre is (7, 6) and radius, $r = 25$

Let $S = (x - 7)^2 + (y - 6)^2 - 625$

$\therefore \quad (S)_{(2,\,-6)} = 25 + 144 - 625 < 0$

$\therefore$ Point lies inside the circle

$$\text{Now, } CP = \sqrt{(7 - 2)^2 + (6 + 6)^2} = 13$$

$\therefore \quad l$ = Minimum distance of P from the circle
$= r - CP = 25 - 13 = 12$

and m = Maximum distance = radius of circle + CP
$= 25 + 13 = 38$

$\therefore \quad l + m = 50$ and $m - l = 26$

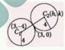
14. (25) : The required circle having radius = 4 and passing through the point (3, 0).

The equation of other circle is $x^2 + y^2 - 6x + 8y + 9 = 0$

$\Rightarrow \quad (x - 3)^2 + (y + 4)^2 = 16$

$\therefore \quad r = 4$ and centre $C_1(3, -4)$

It means circle touches each other externally because their radii are same.



Again the point of contact of both the circles is at (3, 0) which means (3, 0) is mid point of the joining their centres. As the coordinate of centre of other circle is $C_2(h, k)$

$$\therefore \quad \frac{h + 3}{2} = 3 \text{ and } \frac{k - 4}{2} = 0 \Rightarrow h = 3, k = 4$$

$\therefore \quad h^2 + k^2 = 3^2 + 4^2 = 25.$

15. (2) : Equation of circle is $x^2 + y^2 - 4x - 8y + 11 = 0$

$\Rightarrow \quad (x - 2)^2 + (y - 4)^2 = 9$

$\therefore$ Centre is $C(2, 4)$ and radius is $R = 3$

Let CM be the ⊥ from C on the line $5x - 3y + 2 = 0$.

Then $CM = \dfrac{|5(2) - 3(4) + 2|}{\sqrt{5^2 + (-3)^2}} = 0$, which means the line passes through the centre of the circle.

$\therefore$ Intercepted length is diameter of circle and the diameter of circle is 6. $\therefore 3k = 6 \Rightarrow k = 2$.

◆

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YOU ASK WE ANSWER

Do you have a question that you just can't get answered? Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If $f(x)$ is a polynomial of degree <3, prove that

$$\begin{vmatrix} 1 & a & \frac{f(a)}{x-a} \\ 1 & b & \frac{f(b)}{x-b} \\ 1 & c & \frac{f(c)}{x-c} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \frac{f(x)}{(x-a)(x-b)(x-c)}$$

(Ashima Setia, A.P.)

Ans. $\frac{f(x)}{(x-a)(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$

[By partial fractions]

i.e., $f(x) = A(x-b)(x-c)+B(x-a)(x-c) + C(x-a)(x-b)$ ...(i)

Putting $x = a$ in equation (i), we have

$f(a) = A(a-b)(a-c) \Rightarrow A = \frac{f(a)}{(a-b)(a-c)}$

Similarly, putting $x = b$, c in equation (i) respectively, we have $B = \frac{f(b)}{(b-a)(b-c)}$ and $C = \frac{f(c)}{(c-a)(c-b)}$

Hence, we have $\frac{f(x)}{(x-a)(x-b)(x-c)}$

$$= \frac{(c-b)\frac{f(a)}{x-a} - (c-a)\frac{f(b)}{(x-b)} + (b-a)\frac{f(c)}{(x-c)}}{(a-b)(b-c)(c-a)}$$

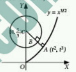
$$= \begin{vmatrix} 1 & a & \frac{f(a)}{x-a} \\ 1 & b & \frac{f(b)}{x-b} \\ 1 & c & \frac{f(c)}{x-c} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$, which is the desired result.

2. Find the shortest distance between the curves $y^2 = x^3$ and $9x^2 + 9y^2 - 30y + 16 = 0$.

(Indrani Das, West Bengal)

Ans. We have $9x^2 + 9y^2 - 30y + 16 = 0$, which is a circle having centre $\equiv (0, 5/3)$ and radius $= \sqrt{\left(\frac{5}{3}\right)^2 - \frac{16}{9}} = 1$.

Let us choose any point on the curve $y^2 = x^3$ as $A(t^2, t^3)$. If B is the point on the circle and nearest to A, then $AB = AC$ – radius



$= \sqrt{(t^2-0)^2 + \left(t^3 - \frac{5}{3}\right)^2} - 1$

Let $f(t) = t^4 + \left(t^3 - \frac{5}{3}\right)^2$

$\Rightarrow f'(t) = 4t^3 + 2\left(t^3 - \frac{5}{3}\right)\cdot 3t^2 = 2t^2\left[2t + 3\left(t^3 - \frac{5}{3}\right)\right]$

$= 2t^2(3t^3 + 2t - 5) = 2t^2(t-1)(3t^2 + 3t + 5)$.

The value of t at which AB attains minima is given by the equation $f'(t) = 0$ gives $t = 0, 1$.

But $t = 0$ is not a point of extrema, since $f'(t)$ does not change sign in the neighbourhood of $t = 0$.

However, $t = 1$ is a point of minima, since $f'(1^-) < 0$ and $f'(1^+) > 0$.

Hence, the minimum value of AB is

$\sqrt{1 + \left(1 - \frac{5}{3}\right)^2} - 1 = 0.2$.

3. Show that $\int_b^a \frac{x^{n-1}[(n-2)x^2 + (n-1)(a+b)x + nab]}{(x+a)^2(x+b)^2}dx = \frac{b^{n-1} - a^{n-1}}{2(a+b)}$.

(Chinmay, Assam)

Ans. Let $I = \int_b^a \frac{x^{n-1}[(n-2)x^2 + (n-1)(a+b)x + nab]}{(x+a)^2(x+b)^2}dx$

$$= \int_a^b \frac{x^{n-1}[n\{x^2 + (a+b)x + ab\} - \{2x^2 + (a+b)x\}]}{(x+a)^2(x+b)^2}dx$$

$$= \int_a^b \frac{nx^{n-1}(x+a)(x+b) - x^n\{2x + (a+b)\}}{(x+a)^2(x+b)^2}dx$$

$$= \int_a^b d\left\{\frac{x^n}{(x+a)(x+b)}\right\} = \left[\frac{x^n}{(x+a)(x+b)}\right]_a^b$$

$$= \frac{b^n}{2b(a+b)} - \frac{a^n}{2a(a+b)} = \frac{b^{n-1} - a^{n-1}}{2(a+b)}.$$

MOCK TEST PAPER 2019
JEE MAIN

Exam Dates : Between 6th to 20th January and between 6th to 20th April 2019

Time : 1 hr 15 min. **Series - 6**

The entire syllabus of Mathematics of WB-JEE is being divided into eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

	Topic	Syllabus In Detail
UNIT NO. 6	Differential calculus	Derivative of the sum, difference, product and quotient of two functions, chain rule, derivatives of polynomial, rational, trigonometric, inverse trigonometric, exponential and logarithmic functions. Derivative of implicit functions, derivative up to order two.
	Trigonometry	Inverse trigonometrical functions and their properties. Height and distance.
	Vector Algebra	Vectors and scalars, addition of vectors, components of a vector in two dimensions and three dimensional space, scalar and vector products, scalar and vector triple product.
	Matrices and Determinants	Matrices as a rectangular array of real numbers, equality of matrices, addition, multiplication by a scalar and product of matrices, transpose of a matrix, determinant of square matrix of order up to three, Properties of determinants, area of triangles using determinants, inverse of a square matrix of order up to three, properties of these matrix operations, diagonal, symmetric and skew-symmetric matrices and their properties, solutions of simultaneous linear equations in two or three variables.

1. If $f(x)=\sin\left\{\frac{\pi}{3}[x]-x^2\right\}$ for $2<x<3$ and $[x]$ denotes the greatest integer less than or equal to x, then $f'\left(\sqrt{\frac{\pi}{3}}\right)$ equal to

(a) $\sqrt{\frac{\pi}{3}}$ (b) $-\sqrt{\frac{\pi}{3}}$ (c) $-\sqrt{\pi}$ (d) $\sqrt{\pi}$

2. If $f(x)=|x-2|$ and $g(x)=f(f(x))$, then for $2<x<4$, $g'(x)=$

(a) -1 (b) 1 (c) 0 (d) 2

3. If $f(x)=|\cos x-\sin x|$, then $f'\left(\frac{\pi}{4}\right)=$

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) 0 (d) None of these

4. If $y=|\cos x|+|\sin x|$, then $\left(\frac{dy}{dx}\right)_{x=2\pi/3}=$

(a) -1 (b) 1

(c) does not exist (d) None of these

5. If $f(x)=|x|^{|\tan x|}$, then $f'\left(-\frac{\pi}{6}\right)=$

(a) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{\frac{2\sqrt{3}}{\pi}-\frac{4}{3}\log\frac{6}{\pi}\right\}$

(b) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{-\frac{2\sqrt{3}}{\pi}-\frac{4}{3}\log\frac{\pi}{6}\right\}$

(c) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{\frac{2\sqrt{3}}{\pi}+\frac{4}{3}\log\frac{6}{\pi}\right\}$

(d) None of these

6. If $y=\sin^{-1}(\sin x)$, then $\left(\frac{dy}{dx}\right)_{x=\pi/2}=$

(a) 1 (b) -1

(c) does not exist (d) None of these

7. Let $F(x)=f(x)g(x)h(x)$ for all real x where $f(x)$, $g(x)$ and $h(x)$ are differentiable functions. At some point x_0, if $F'(x_0)=21F(x_0)$, $f'(x_0)=4f(x_0)$, $g'(x_0)=-7g(x_0)$ and $h'(x_0)=\lambda h(x_0)$, then $\lambda=$

(a) 12 (b) -12 (c) 24 (d) -24

By : Sankar Ghosh, S.G.M.C, Mob. 09831244397.

8. The derivative of $\operatorname{cosec}^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x=\frac{1}{2}$ is

(a) -4 (b) 4 (c) -1 (d) 2

9. If $f(x)=\cos^{-1}\left\{\frac{1-(\log_e x)^2}{1+(\log_e x)^2}\right\}$, then $f'\left(\frac{1}{e}\right)=$

(a) e (b) $-e$ (c) 1 (d) $2e$

10. If $\frac{dx}{dy}=\left(\frac{dy}{dx}\right)^{-1}$, then $\frac{d^2y}{dx^2}=$

(a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-1}$

(c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

11. If a curve is represented parametrically by the equations, $x=4t^3+3$ and $y=4+3t^4$ and $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^n$ is constant, then the value of n is

(a) 3 (b) 4 (c) 5 (d) 6

12. If $y=x+e^x$, then $\frac{d^2x}{dy^2}$ is

(a) e^x (b) $-\frac{e^x}{(1+e^x)^3}$

(c) $-\frac{e^x}{(1+e^x)^2}$ (d) $\frac{1}{(1+e^x)^2}$

13. If $x=\cos\theta$ and $y=\sin 5\theta$, then $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=$

(a) $-5y$ (b) $5y$ (c) 25 (d) $-25y$

14. The roots of the equation,

$17x^2+17x\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)-10=0$ is

(a) $\frac{10}{17}$ (b) -1 (c) $-\frac{7}{17}$ (d) 1

15. The value of $\cos\left\{\tan^{-1}\left(\tan\frac{15\pi}{4}\right)\right\}$, is

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$

16. If $\log_2 x \geq 0$, then

$\log_{\frac{1}{\pi}}\left\{\sin^{-1}\frac{2x}{1+x^2}+2\tan^{-1}x\right\}$ is equal to

(a) $\log_{\frac{1}{\pi}}(4\tan^{-1}x)$ (b) 0

(c) -1 (d) 1

17. The number of solutions of

$\sin\left\{\sin^{-1}(\log_{1/2}x)\right\}+2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2}-\frac{3}{2}\right)\right\}\right|=0$, is

(a) 1 (b) 2

(c) 3 (d) None of these

18. If $\left|\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right|<\frac{\pi}{3}$, then x belongs to the interval

(a) $\left[-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right]$ (b) $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

(c) $\left(0,\frac{1}{\sqrt{3}}\right)$ (d) None of these

19. If $f(x)=\cos^{-1}x+\cos^{-1}\left\{\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right\}$, then $f\left(\frac{2}{3}\right)=$

(a) $\frac{\pi}{3}$ (b) $2\cos^{-1}\frac{2}{3}-\frac{\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $2\cos^{-1}\frac{2}{3}$

20. If $\cot(\cos^{-1}x)=\sec\left\{\tan^{-1}\left(\frac{a}{\sqrt{b^2-a^2}}\right)\right\}$, then x equals

(a) $\frac{b}{\sqrt{2b^2-a^2}}$ (b) $\frac{a}{\sqrt{2b^2-a^2}}$

(c) $\frac{\sqrt{b^2-a^2}}{a}$ (d) $\frac{\sqrt{b^2-a^2}}{b}$

21. If $(\sin^{-1}x+\sin^{-1}y)(\sin^{-1}z+\sin^{-1}w)=\pi^2$ and $n_1, n_2, n_3, n_4 \in N$, then value of $\begin{vmatrix} x^{n_1} & y^{n_2} \\ z^{n_3} & w^{n_4} \end{vmatrix}$ cannot be equal to

(a) -2 (b) 0 (c) 1 (d) 2

22. From the top of a cliff 300 metres high, the top of a tower was observed at an angle of depression 30° and from the foot of the tower the top of the cliff was observed at an angle of elevation 45°. The height of the tower is

(a) $50(3-\sqrt{3})$ m (b) $200(3-\sqrt{3})$ m

(c) $100(3-\sqrt{3})$ m (d) None of these

23. Three vertical poles of heights h_1, h_2 and h_3 at vertices A, B, and C of a ΔABC subtend angles α, β and γ respectively at the circumcentre of the triangle. If $\cot\alpha$, $\cot\beta$ and $\cot\gamma$ are in A.P., then h_1, h_2, h_3 are in

(a) A.P. (b) G.P.
(c) H.P. (d) None of these

24. Let $\overline{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $\overline{AC} = \hat{i} - \hat{j} + 3\hat{k}$ and a point P on the line segment BC is equidistant from AB and AC, then $\overline{AP}$ is

(a) $2\hat{i} - \hat{k}$ (b) $\hat{i} - 2\hat{k}$ (c) $2\hat{i} + \hat{k}$ (d) $\hat{i} - \hat{j}$

25. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$. If $\vec{b}$ is a vector satisfying $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$, then $\vec{b}$ is

(a) $\frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$ (b) $\frac{1}{3}(5\hat{i} - 2\hat{j} - 2\hat{k})$
(c) $3\hat{i} - \hat{j} - \hat{k}$ (d) None of these

26. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 2 (d) 3

27. Let $\vec{a}$ and $\vec{b}$ are two vectors inclined at an angle of 60°. If $|\vec{a}| = |\vec{b}| = 2$, then the angle between $\vec{a}$ and $\vec{a} + \vec{b}$ is

(a) 30° (b) 60° (c) 45° (d) 90°

28. Consider the system of equations in x, y, z
$x\sin 3\theta - y + z = 0$, $x\cos 2\theta + 4y + 3z = 0$,
$2x + 7y + 7z = 0$
If this has a non-trivial solution, then for any integer n, values of θ are given by

(a) $\left(n + \frac{(-1)^n}{3}\right)\pi$ (b) $\left(n + \frac{(-1)^n}{4}\right)\pi$

(c) $\left(n + \frac{(-1)^n}{6}\right)\pi$ (d) $\frac{n\pi}{2}$

29. Let $D = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$; $0 \le \theta \le 2\pi$,

then $D \in$

(a) $\{\phi\}$ (b) $(0, \infty)$ (c) $[2, 4]$ (d) $[2, \infty)$

30. If $A = \int_1^{\sin\theta} \frac{t}{1+t^2}\,dt$ and $B = \int_1^{\text{cosec}\,\theta} \frac{1}{t(1+t^2)}\,dt$, then value of the determinant $\begin{vmatrix} A & A^2 & B \\ e^{A+B} & B^2 & -1 \\ 1 & A^2 + B^2 & -1 \end{vmatrix}$ is

(a) $\sin\theta$ (b) $\text{cosec}\,\theta$ (c) 0 (d) 1

SOLUTIONS

1. (b) : For $2 < x < 3$, the value of $[x]$ is 2

$\therefore \ f(x) = \sin\left(\frac{2\pi}{3} - x^2\right)$

$\Rightarrow \ f'(x) = -2x\cos\left(\frac{2\pi}{3} - x^2\right)$

$\Rightarrow \ f'\left(\sqrt{\frac{\pi}{3}}\right) = -2\sqrt{\frac{\pi}{3}}\cos\frac{\pi}{3} = -\sqrt{\frac{\pi}{3}}$

2. (a) : $f(x) = \begin{cases} x - 2, & \text{when } 2 \le x < 4 \\ -(x-2), & \text{when } x < 2 \end{cases}$

Also, $g(x) = f(f(x))$

$\Rightarrow \ g(x) = \begin{cases} f(x-2), & \text{when } 2 \le x < 4 \\ f(-(x-2)), & \text{when } x < 2 \end{cases}$

$\Rightarrow \ g(x) = \begin{cases} -(x-2-2), & \text{when } 2 \le x < 4 \\ -(-x+2-2), & \text{when } 0 < x < 2 \end{cases}$

$\Rightarrow \ g'(x) = \begin{cases} -1, & 2 < x < 4 \\ 1, & 0 < x < 2 \end{cases}$

3. (d) : $f(x) = \begin{cases} \cos x - \sin x, & \text{when } 0 < x < \frac{\pi}{4} \\ \sin x - \cos x, & \text{when } \frac{\pi}{4} \le x < \frac{\pi}{2} \end{cases}$

Now, $Lf'\left(\frac{\pi}{4}\right) = \left\{\frac{d}{dx}(\cos x - \sin x)\right\}_{x=\frac{\pi}{4}} = -\sqrt{2}$

and $Rf'\left(\frac{\pi}{4}\right) = \left\{\frac{d}{dx}(\sin x - \cos x)\right\}_{x=\frac{\pi}{4}} = \sqrt{2}$

Thus, $f'\left(\frac{\pi}{4}\right)$ does not exist.

4. (c) : In the neighbourhood of $x = \frac{2\pi}{3}$, we have $\cos x < 0$ and $\sin x > 0$
$\Rightarrow \ |\cos x| = -\cos x$ and $|\sin x| = \sin x$
Given $y = |\cos x| + |\sin x| \quad \therefore \quad y = -\cos x + \sin x$

Now, R.H.D (at $x = \frac{2\pi}{3}$)

$$= \lim_{h\to 0} \frac{-\cos\left(\frac{2\pi}{3}+h\right)+\sin\left(\frac{2\pi}{3}+h\right)-\frac{\sqrt{3}+1}{2}}{h} = \frac{\sqrt{3}-1}{2}$$

and L.H.D. $\left(\text{at } x = \frac{2\pi}{3}\right)$

$$= \lim_{h\to 0} \frac{-\cos\left(\frac{2\pi}{3}-h\right)+\sin\left(\frac{2\pi}{3}-h\right)-\frac{1+\sqrt{3}}{2}}{h} = \frac{1-\sqrt{3}}{2}$$

Thus, $f'\left(\frac{2\pi}{3}\right)$ does not exist.

5. (b) : We have $f(x) = |x|^{|\tan x|}$

$\Rightarrow f(x) = (-x)^{-\tan x}$ [$\because$ In the neighbourhood of $x = -\frac{\pi}{6}$, $|x| = -x$ and $|\tan x| = -\tan x$]

$\Rightarrow \log f(x) = -\tan x \log(-x)$

$$\Rightarrow \frac{1}{f(x)} \cdot f'(x) = \left[-\sec^2 x \cdot \log(-x) - \frac{\tan x}{x}\right]$$

$$\Rightarrow f'(x) = (-x)^{-\tan x}\left[-\sec^2 x \cdot \log(-x) - \frac{\tan x}{x}\right]$$

$$\Rightarrow f'\left(-\frac{\pi}{6}\right) = \left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{-\frac{4}{3}\log\frac{\pi}{6} - \frac{2\sqrt{3}}{\pi}\right\}$$

6. (c) : We have,

$$y = \sin^{-1}(\sin x) = \begin{cases} x & , -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ \pi - x & , \frac{\pi}{2} \le x \le \frac{3\pi}{2} \\ x - 2\pi & , \frac{3\pi}{2} \le x \le 2\pi \end{cases}$$

$\therefore$ At $x = \frac{\pi}{2}$, L.H.D. = 1 and R.H.D. = −1.

Hence $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}}$ does not exist.

7. (c) : Given that $F(x) = f(x)\, g(x)\, h(x)$

$\Rightarrow \log F(x) = \log f(x) + \log g(x) + \log h(x)$

$$\Rightarrow \frac{F'(x)}{F(x)} = \frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)} + \frac{h'(x)}{h(x)}$$

$$\Rightarrow \frac{F'(x_0)}{F(x_0)} = \frac{f'(x_0)}{f(x_0)} + \frac{g'(x_0)}{g(x_0)} + \frac{h'(x_0)}{h(x_0)}$$

$\Rightarrow 21 = 4 - 7 + \lambda \Rightarrow \lambda = 24$

8. (a) : Let $u = \operatorname{cosec}^{-1}\left(\frac{1}{2x^2-1}\right)$

$$\Rightarrow u = \sin^{-1}(2x^2-1) = \frac{\pi}{2} - \cos^{-1}(2x^2-1)$$

$$\Rightarrow u = \begin{cases} \frac{\pi}{2} - 2\cos^{-1} x & , \text{if } 0 \le x \le 1 \\ -\frac{3\pi}{2} + 2\cos^{-1} x & , \text{if } -1 \le x \le 0 \end{cases}$$

$$\Rightarrow \frac{du}{dx} = \begin{cases} \frac{2}{\sqrt{1-x^2}} & , \text{if } 0 < x < 1 \\ -\frac{2}{\sqrt{1-x^2}} & , \text{if } -1 < x < 0 \end{cases}$$

Also, $v = \sqrt{1-x^2}$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{-x}{\sqrt{1-x^2}} \ \forall\, x \in (-1, 1)$$

$$\Rightarrow \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \begin{cases} -\frac{2}{x} & , \text{if } 0 < x < 1 \\ \frac{2}{x} & , \text{if } -1 < x < 0 \end{cases}$$

$$\Rightarrow \left(\frac{du}{dv}\right)_{x=\frac{1}{2}} = -4$$

9. (b) : In the neighbourhood of $x = \frac{1}{e}$, $\log_e x < 0$

$$\therefore f(x) = \cos^{-1}\left\{\frac{1-(\log x)^2}{1+(\log x)^2}\right\} = -2\tan^{-1}(\log_e x)$$

$$\Rightarrow f'(x) = -\frac{2}{1+(\log_e x)^2} \times \frac{1}{x} \Rightarrow f'\left(\frac{1}{e}\right) = -e$$

10. (d) : Given, $\frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$

$$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\} \times \frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-2} \frac{d}{dx}\left(\frac{dy}{dx}\right) \times \frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-3}\left(\frac{d^2y}{dx^2}\right)$$

11. (c) : We have, $x = 4t^3 + 3$ and $y = 4 + 3t^4$

$$\Rightarrow \quad \frac{dx}{dt} = 12t^2 \text{ and } \frac{dy}{dt} = 12t^3 \quad \Rightarrow \quad \frac{dx}{dy} = \frac{1}{t}$$

Now, $\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{dx}{dy}\right) = \dfrac{d}{dy}\left(\dfrac{1}{t}\right) = \dfrac{d}{dt}\left(\dfrac{1}{t}\right)\dfrac{dt}{dy}$

$$= -\frac{1}{t^2} \times \frac{1}{12t^3} = -\frac{1}{12t^5}$$

$$\therefore \quad \frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^n = -\frac{1}{12t^5} \times t^n = -\frac{1}{12} t^{n-5} \qquad \text{...(i)}$$

Clearly, (i) will be constant if $n = 5$.

12. (b) : We have, $y = x + e^x \Rightarrow \dfrac{dx}{dy} = \dfrac{1}{(1+e^x)}$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\frac{1}{1+e^x}\right\}$$

$$= \frac{d}{dx}\left\{\frac{1}{1+e^x}\right\}\frac{dx}{dy} = -\frac{e^x}{(1+e^x)^2} \times \frac{1}{(1+e^x)} = -\frac{e^x}{(1+e^x)^3}$$

13. (d) : We have, $x = \cos\theta$ and $y = \sin 5\theta$

$$\therefore \frac{dx}{d\theta} = -\sin\theta \text{ and } \frac{dy}{d\theta} = 5\cos 5\theta \Rightarrow \frac{dy}{dx} = -\frac{5\cos 5\theta}{\sin\theta}$$

Now, $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dx}\left(-\dfrac{5\cos 5\theta}{\sin\theta}\right)$

$$= -5\frac{d}{d\theta}\left(\frac{\cos 5\theta}{\sin\theta}\right)\frac{d\theta}{dx} = \frac{-25\sin 5\theta \sin\theta - 5\cos\theta\cos 5\theta}{\sin^3\theta}$$

Thus, $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx}$

$$= \sin^2\theta \times \frac{-25\sin 5\theta\sin\theta - 5\cos\theta\cos 5\theta}{\sin^3\theta} - \cos\theta\left(-\frac{5\cos 5\theta}{\sin\theta}\right)$$

$$= \frac{-25\sin 5\theta\sin\theta}{\sin\theta} = -25\sin 5\theta = -25y$$

14. (d) : We have,

$$17x^2 + 17x\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) - 10 = 0 \qquad \text{...(i)}$$

Consider, $\tan\left(2\tan^{-1}\dfrac{1}{5} - \dfrac{\pi}{4}\right) = \tan\left(\tan^{-1}\dfrac{2/5}{1-1/25} - \tan^{-1}1\right)$

$$= \tan\left(\tan^{-1}\frac{5}{12} - \tan^{-1}1\right) = -\frac{7}{17}$$

Thus, (i) becomes $17x^2 - 7x - 10 = 0$

$$\Rightarrow \quad (x-1)(17x+10) = 0 \Rightarrow \quad x = 1, -\frac{10}{17}$$

15. (a) : We have, $\cos\left\{\tan^{-1}\left(\tan\dfrac{15\pi}{4}\right)\right\}$

$$= \cos\left\{\tan^{-1}\left(\tan\left(4\pi - \frac{\pi}{4}\right)\right)\right\}$$

$$= \cos\left\{\tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right)\right\} = \cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

16. (c) : We have, $\log_2 x \geq 0 \Rightarrow x \geq 1$

For $x \geq 1$, we have

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan^{-1}x$$

$$\therefore \quad \log_{\frac{1}{\pi}}\left[\sin^{-1}\frac{2x}{1+x^2} + 2\tan^{-1}x\right]$$

$$= \log_{\frac{1}{\pi}}\left[\pi - 2\tan^{-1}x + 2\tan^{-1}x\right] = \log_{\frac{1}{\pi}}\pi = -1$$

17. (d) : We have,

$$\sin\left\{\sin^{-1}\left(\log_{1/2}x\right)\right\} + 2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2} - \frac{3}{2}\right)\right\}\right| = 0$$

The two terms on L.H.S. of the given equation are meaningful, if $-1 \leq \log_{1/2} x \leq 1$ and $-1 \leq \dfrac{x}{2} - \dfrac{3}{2} \leq 1$

Now, $1 \leq x \leq 2 \Rightarrow \quad \dfrac{1}{2} \leq \dfrac{x}{2} \leq 1$

$$\Rightarrow \quad -\frac{\pi}{6} \leq \sin^{-1}\left(\frac{x}{2} - \frac{3}{2}\right) \leq 0$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} \leq \cos\left\{\sin^{-1}\left(\frac{x}{2} - \frac{3}{2}\right)\right\} \leq 1$$

$$\Rightarrow \quad \sqrt{3} \leq 2\cos\left\{\sin^{-1}\left(\frac{x}{2} - \frac{3}{2}\right)\right\} \leq 2$$

$$\Rightarrow \quad \sqrt{3} \leq 2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2} - \frac{3}{2}\right)\right\}\right| \leq 2 \qquad \text{...(i)}$$

Again $\quad 1 \leq x \leq 2$

$$\Rightarrow \quad -1 \leq \log_{1/2} x \leq 0 \quad \Rightarrow \quad -\frac{\pi}{2} \leq \sin^{-1}\left(\log_{1/2} x\right) \leq 0$$

$$\Rightarrow \quad -1 \leq \sin\left\{\sin^{-1}\left(\log_{1/2} x\right)\right\} \leq 0 \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$\sqrt{3} - 1 \leq \sin\left\{\sin^{-1}\left(\log_{1/2} x\right)\right\} + 2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2} - \frac{3}{2}\right)\right\}\right| \leq 2$$

$$\Rightarrow \quad \sin\left\{\sin^{-1}\left(\log_{1/2} x\right)\right\} + 2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2} - \frac{3}{2}\right)\right\}\right| \neq 0$$

Hence the given equation has no solution.

18. (b) : We have, $\left|\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right| < \frac{\pi}{3}$

$\Rightarrow \quad 0 < \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) < \frac{\pi}{3} \qquad \left[\because \cos^{-1} x \in [0, \pi]\right]$

$\Rightarrow \quad \frac{1}{2} < \frac{1-x^2}{1+x^2} < 1 \Rightarrow \quad 1 + x^2 < 2 - 2x^2 < 2 + 2x^2$

$\Rightarrow \quad 1 + x^2 < 2 - 2x^2$ and $2 - 2x^2 < 2 + 2x^2$

$\Rightarrow \quad 3x^2 - 1 < 0$ and $4x^2 > 0 \Rightarrow -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

19. (a) : We know that,

$$\cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\}$$

$$= \begin{cases} \cos^{-1} x - \cos^{-1} y, -1 \le x, y \le 1 \text{ and } x \le y \\ \cos^{-1} y - \cos^{-1} x, \ -1 \le y \le 0, 0 \le x \le 1 \text{ and } y \le x \end{cases}$$

Consider $\cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right\}$

$$= \cos^{-1}\left\{x \times \frac{1}{2} + \sqrt{1-x^2}\sqrt{1-\frac{1}{4}}\right\}$$

$$= \begin{cases} \cos^{-1} x - \cos^{-1}\frac{1}{2}, -1 \le x \le \frac{1}{2} \\ \cos^{-1}\frac{1}{2} - \cos^{-1} x, \ \frac{1}{2} \le x \le 1 \end{cases}$$

$$\therefore \ f(x) = \begin{cases} 2\cos^{-1} x - \cos^{-1}\frac{1}{2}, -1 \le x \le \frac{1}{2} \\ \cos^{-1}\frac{1}{2} \qquad , \frac{1}{2} \le x \le 1 \end{cases}$$

$\Rightarrow \quad f\left(\frac{2}{3}\right) = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

20. (a) : Consider,

$$\cot(\cos^{-1} x) = \sec\left\{\tan^{-1}\left(\frac{a}{\sqrt{b^2-a^2}}\right)\right\}$$

$\Rightarrow \cot\left\{\cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right\} = \sec\left\{\sec^{-1}\left(\frac{b}{\sqrt{b^2-a^2}}\right)\right\}$

$\Rightarrow \quad \frac{x}{\sqrt{1-x^2}} = \frac{b}{\sqrt{b^2-a^2}} \Rightarrow \quad \frac{x^2}{1-x^2} = \frac{b^2}{b^2-a^2}$

$\Rightarrow \quad \frac{1-x^2}{x^2} = \frac{b^2-a^2}{b^2} \Rightarrow \quad \frac{1}{x^2} - 1 = 1 - \frac{a^2}{b^2}$

$\Rightarrow \quad \frac{1}{x^2} = \frac{2b^2-a^2}{b^2} \Rightarrow x^2 = \frac{b^2}{2b^2-a^2}$

21. (c) : The minimum and maximum value of $\sin^{-1}x + \sin^{-1}y$ are $-\pi$ and π respectively.

$\therefore \quad (\sin^{-1}x + \sin^{-1}y)(\sin^{-1}z + \sin^{-1}w) = \pi^2$

$\Rightarrow \left(\sin^{-1} x = \sin^{-1} y = \frac{\pi}{2} = \sin^{-1} z = \sin^{-1} w\right)$

or $\left(\sin^{-1} x = \sin^{-1} y = -\frac{\pi}{2} = \sin^{-1} z = \sin^{-1} w\right)$

$\Rightarrow \quad x = y = z = w = 1$ or $x = y = z = w = -1$

When $x = y = z = w = 1$, then $\begin{vmatrix} x^{n_1} & y^{n_2} \\ z^{n_3} & w^{n_4} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$

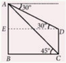
When $x = y = z = w = -1$

Then, $\begin{vmatrix} x^{n_1} & y^{n_2} \\ z^{n_3} & w^{n_4} \end{vmatrix} = \begin{vmatrix} (-1)^{n_1} & (-1)^{n_2} \\ (-1)^{n_3} & (-1)^{n_4} \end{vmatrix}$

$= (-1)^{n_1+n_4} - (-1)^{n_2+n_3} = 0, 2, -2$

Hence, $\begin{vmatrix} x^{n_1} & y^{n_2} \\ z^{n_3} & w^{n_4} \end{vmatrix}$ cannot be equal to 1.

22. (c) : Let the height of the tower be $DC = x$ and height of the cliff be $AB = 300$ m (Given)



$\therefore$ From ΔABC, we get

$\frac{AB}{BC} = \tan 45° \Rightarrow \frac{AB}{BC} = 1$

$\Rightarrow \quad AB = BC$

From the figure, it is clear that $BC = ED = 300$

$(\because AB = 300)$

Now from ΔAED, we have

$\frac{AE}{ED} = \tan 30° \Rightarrow \frac{AB - EB}{ED} = \frac{1}{\sqrt{3}} \Rightarrow \frac{300 - x}{300} = \frac{1}{\sqrt{3}}$

$[\because EB = DC = x]$

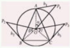
$\therefore \quad 300\sqrt{3} = 300 + x\sqrt{3} \Rightarrow x = 100(3 - \sqrt{3})$

Thus, the height of the tower is $100(3 - \sqrt{3})$.

23. (c) : Let R be the circum-radius and O be the circumcentre of ΔABC.

In ΔOAP_1, we have

$\tan\alpha = \frac{AP_1}{OA} \Rightarrow \tan\alpha = \frac{h_1}{R} \Rightarrow \frac{R}{h_1} = \cot\alpha$



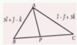
Similarly, $\cot\beta = \frac{R}{h_2}$; $\cot\gamma = \frac{R}{h_3}$

But $\cot\alpha$, $\cot\beta$, $\cot\gamma$ are in A.P.

$\therefore \quad \frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{h_3}$ are in A.P.

Thus h_1, h_2, h_3 are in H.P.

24. (c) : Given that the point P is on BC, such that it is equidistant from AB and AC. Therefore the point P is on the bisector of the angle $\angle BAC$. The vector along the bisector of $\angle BAC$ is



$$\frac{\overline{AB}}{|\overline{AB}|}+\frac{\overline{AC}}{|\overline{AC}|}=\frac{1}{\sqrt{11}}(4\hat{i}+2\hat{k})=\frac{2}{\sqrt{11}}(2\hat{i}+\hat{k})$$

Let $\overline{AP}=\lambda(2\hat{i}+\hat{k})$

$\therefore\quad \overline{BP}=\overline{AP}-\overline{AB}$

$=\lambda(2\hat{i}+\hat{k})-(3\hat{i}+\hat{j}-\hat{k})$

$\Rightarrow\quad \overline{BP}=(2\lambda-3)\hat{i}-\hat{j}+(\lambda+1)\hat{k}$

Also, $\overline{BC}=\overline{AC}-\overline{AB}=-2\hat{i}-2\hat{j}+4\hat{k}$

Since $\overline{BP}\parallel\overline{BC}$, therefore $\overline{BP}=t\overline{BC}$

$\Rightarrow\quad (2\lambda-3)\hat{i}-\hat{j}+(\lambda+1)\hat{k}=t(-2\hat{i}-2\hat{j}+4\hat{k})$

$\Rightarrow\quad 2\lambda-3=-2t,\ -1=-2t,\ \lambda+1=4t\Rightarrow\lambda=1,\ t=\frac{1}{2}$

$\therefore\quad \overline{AP}=2\hat{i}+\hat{k}$

25. (a) : Let $\vec{b}=x\hat{i}+y\hat{j}+z\hat{k}$.

Then $\vec{a}\cdot\vec{b}=3$ (Given) $\Rightarrow x+y+z=3$ ...(i)

and $\vec{a}\times\vec{b}=\vec{c}\Rightarrow\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&1\\x&y&z\end{vmatrix}=\hat{j}-\hat{k}$

$\Rightarrow\quad (z-y)\hat{i}+(x-z)\hat{j}+(y-x)\hat{k}=\hat{j}-\hat{k}$

$\Rightarrow\quad z-y=0,\ x-z=1$ and $y-x=-1$ ...(ii)

Now solving (i) and (ii) we get

$x=\frac{5}{3},\ y=\frac{2}{3}$ and $z=\frac{2}{3}\quad\therefore\quad\vec{b}=\frac{1}{3}(5\hat{i}+2\hat{j}+2\hat{k})$

26. (b) : Here, $\vec{a}=2\hat{i}+\hat{j}-2\hat{k},\ \vec{b}=\hat{i}+\hat{j}$

$\therefore\ \vec{a}\times\vec{b}=2\hat{i}-2\hat{j}+\hat{k}$

$\therefore\ |(\vec{a}\times\vec{b})\times\vec{c}|=|\vec{a}\times\vec{b}||\vec{c}|\sin 30°=\frac{3}{2}|\vec{c}|$

Now, $|\vec{c}-\vec{a}|=2\sqrt{2}\Rightarrow|\vec{c}-\vec{a}|^2=8$

$\Rightarrow\quad |\vec{c}|^2+|\vec{a}|^2-2(\vec{a}\cdot\vec{c})=8\ \Rightarrow|\vec{c}|=1$

Hence, $|(\vec{a}\times\vec{b})\times\vec{c}|=\frac{3}{2}$

27. (a) : Given, $|\vec{a}|=|\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos 60°=2$

Now, $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2(\vec{a}\cdot\vec{b})=4+4+4$

$\Rightarrow\quad |\vec{a}+\vec{b}|=2\sqrt{3}$

Let θ be the angle between $\vec{a}$ and $\vec{a}+\vec{b}$

$\therefore\quad \cos\theta=\frac{\vec{a}\cdot(\vec{a}+\vec{b})}{|\vec{a}||\vec{a}+\vec{b}|}=\frac{\vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{a}+\vec{b}|}=\frac{4+2}{2\times2\sqrt{3}}=\frac{\sqrt{3}}{2}$

$\Rightarrow\quad \theta=30°$

28. (c) : The given system of equations has a non-trivial solution.

$\therefore\quad\begin{vmatrix}\sin 3\theta&-1&1\\\cos 2\theta&4&3\\2&7&7\end{vmatrix}=0$

$\Rightarrow\quad 7\sin 3\theta+14\cos 2\theta-14=0$

$\Rightarrow\quad 3\sin\theta-4\sin^3\theta+2-4\sin^2\theta-2=0$

$\Rightarrow\quad \sin\theta=0$ or $\sin\theta=\frac{1}{2}$

$\Rightarrow\quad \theta=n\pi$ or $\theta=n\pi+(-1)^n\frac{\pi}{6}$

29. (c) : $D=\begin{vmatrix}1&\sin\theta&1\\-\sin\theta&1&\sin\theta\\-1&-\sin\theta&1\end{vmatrix}$

Applying $C_1\rightarrow C_1+C_3$, we have

$$\begin{vmatrix}2&\sin\theta&1\\0&1&\sin\theta\\0&-\sin\theta&1\end{vmatrix}=2(1+\sin^2\theta)$$

Now, $0\le\sin^2\theta\le1\Rightarrow2\le2(1+\sin^2\theta)\le4$

$\Rightarrow\quad D\in[2,4]$

30. (c) : Here,

$$A+B=\int_1^{\sin\theta}\frac{t}{1+t^2}dt+\int_1^{\operatorname{cosec}\theta}\frac{1}{t(1+t^2)}dt$$

$$=\int_1^{\sin\theta}\frac{t}{1+t^2}dt+\int_1^{\sin\theta}-\frac{z}{1+z^2}dz=0$$

[Put $z=\frac{1}{t}\Rightarrow dt=-t^2dz$]

Hence, $A+B=0\Rightarrow B=-A$

$$\therefore\quad\begin{vmatrix}A&A^2&B\\e^{A+B}&B^2&-1\\1&A^2+B^2&-1\end{vmatrix}=\begin{vmatrix}A&A^2&-A\\1&A^2&-1\\1&2A^2&-1\end{vmatrix}$$

$$=-\begin{vmatrix}A&A^2&A\\1&A^2&1\\1&2A^2&1\end{vmatrix}=0$$

◆◆

BRAIN@WORK

Indefinite Integrals

STANDARD FORMULAE

- $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x+\sqrt{x^2+a^2}\right|+c$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right|+c$
- $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c$
- $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$
- $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+c$
- $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
- $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|+c$
- $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+c$
- $\int \sqrt{a^2-x^2}\;dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

FORMULAE AND CONCEPTS AT A GLANCE

- $\int af(x)dx = a\int f(x)dx$ (Here 'a' is a constant)
- $\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$
- If $\int f(u)du = F(u) + c$, then $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c,\ a \neq 0$
- For $\int f(g(x))\,g'(x)dx$, put $g(x) = t$, provided $\int f(t)dt$ exists.
- For terms of the form $x^2 + a^2$ or $\sqrt{x^2+a^2}$, put $x = a\tan\theta$ or $a\cot\theta$
- For terms of the form $x^2 - a^2$ or $\sqrt{x^2-a^2}$, put $x = a\sec\theta$ or $a\,\text{cosec}\,\theta$
- For terms of the form $a^2 - x^2$ or $\sqrt{a^2-x^2}$, put $x = a\sin\theta$ or $a\cos\theta$
- If both $\sqrt{a+x}, \sqrt{a-x}$ are present, then put $x = a\cos\theta$
- For the type $\sqrt{(x-a)(b-x)}$, put $x = a\cos^2\theta + b\sin^2\theta$
- For the type $(\sqrt{x^2+a^2} \pm x)^n$ or $(x \pm \sqrt{x^2-a^2})^n$, put the expression within the bracket $= t$.
- For the type $(x+a)^{-1-\frac{1}{n}}(x+b)^{-1+\frac{1}{n}}$ or $\left(\frac{x+b}{x+a}\right)^{\frac{1}{n}-1}\frac{1}{(x+a)^2}$ $(n \in N, n > 1)$, put $\frac{x+b}{x+a} = t$
- For $\frac{1}{(x+a)^{n_1}(x+b)^{n_2}}$, $n_1, n_2 \in N$ (and > 1), again put $(x+a) = t(x+b)$.
- $\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx.$
  For selecting 'u' *i.e.*, first function use ILATE.
- $\int e^x\left[f(x)+f'(x)\right]dx = e^x f(x) + c.$
- For proper fractions of the type $P(x)/Q(x)$ use partial fractions
- $\int f(x,(ax+b)^{1/n})dx = \int f\left(\frac{t^n-b}{a}, t\right)\frac{nt^{n-1}}{a}dt$ by writing $t^n = ax+b$.
- For $\int \frac{dx}{(x-k)^r\sqrt{ax^2+bx+c}}$, we substitute, $x - k = 1/t$ leading to $-\int \frac{t^{r-1}dt}{\sqrt{At^2+Bt+c}}$.
- For $\frac{1}{(Ax^2+B)\sqrt{Cx^2+D}}$, we first put $x = 1/t$, and then substitute $C + Dt^2 = u^2$ to get the form $\int \frac{du}{u^2 \pm a^2}$.

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- For $\int \frac{(ax^2+bx+c)dx}{(dx+e)\sqrt{fx^2+gx+h}}$.

  Put $ax^2+bx+c = A_1(dx+e)(2fx+g) + B_1(dx+e) + C_1$ and proceed.

- For $\int \frac{p\ \cos x + q\ \sin x + r}{a\ \cos x + b\ \sin x + c}dx$, express numerator as l (Denominator) + m (diff. of denominator) + n, where $l = \frac{ap+bq}{a^2+b^2}$, $m = \frac{pb-qa}{a^2+b^2}$, $n = r - lc$

PROBLEMS

1. If n is an odd positive integer, then $\int |x^n|dx$ is equal to

(a) $\left|\frac{x^{n+1}}{n+1}\right| + c$ (b) $\frac{x^{n+1}}{n+1} + c$

(c) $\frac{|x^n|x}{n+1} + c$ (d) $\frac{-|x|^n}{n+1} + c$

2. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equal to

(a) $\frac{\sqrt{2x^4-2x^2+1}}{x^2} + c$ (b) $\frac{\sqrt{2x^4-2x^2+1}}{x^3} + c$

(c) $\frac{\sqrt{2x^4-2x^2+1}}{x} + c$ (d) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2} + c$

3. If $I_{m,n} = \int \cos^m x \cos nx dx$, then the value of $(m+n)I_{m,n} - m\, I_{m-1,\,n-1}$ $(m, n \in N)$ is equal to

(a) $\frac{\cos^m x \sin nx}{n} + c$ (b) $\cos^m x \sin nx + c$

(c) $\frac{\cos^m x \cos nx}{n} + c$ (d) $-\cos^m x \cos nx + c$

4. If $\int \frac{(1-x^7)}{x(1+x^7)}dx = P\log|x| + Q\log|x^7+1| + c$, then

(a) $7P + 2Q = 1$ (b) $7P - 2Q = 1$

(c) $7P + 2Q = 0$ (d) $2P + 7Q = 0$

5. If $\int e^{ax}\cos bx dx = \frac{e^{2x}}{29}f(x) + c$, then

(a) $\frac{f''(x)}{f(x)} = 24$ (b) $\frac{f''(x)}{f(x)} = 25$

(c) Range of $f(x)$ is $(-\sqrt{29}, \sqrt{29})$

(d) $\frac{f(x)}{f'(x)} = e^{\cos bx}(a\sin x)$

6. $\int \frac{\cos x(1+4\cos 2x)}{\sin x + 4\sin x\cos^2 x}dx$ is equal to

(a) $\frac{1}{2}\ln|\sin^2 x + \sin^2 2x| + c$

(b) $\frac{1}{2}\ln|\sin x| + \frac{1}{2}\ln|\sin x + 4\sin x\cos^2 x| + c$

(c) $\frac{1}{2}\ln|\sin^2 x - \sin^2 2x| + c$

(d) $\frac{1}{2}\ln|\sin x| - \ln|\sin x + 4\sin x\cos^2 x| + c$

7. The value of $\int \frac{(x^2+1)}{(x^4-x^2+1)\cot^{-1}\left(x-\frac{1}{x}\right)}dx$ will be

(a) $\ln\left|\cot^{-1}\left(x-\frac{1}{x}\right)\right| + c$ (b) $-\ln\left|\cot^{-1}\left(x-\frac{1}{x}\right)\right| + c$

(c) $\ln\left|\cot^{-1}\left(x+\frac{1}{x}\right)\right| + c$ (d) None of these

8. If $\int \frac{dx}{\cos^3 x - \sin^3 x} = A\tan^{-1}(\sin x + \cos x) + B\ln(f(x)) + C$ then A is equal to

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{5}$

9. $\int \frac{dx}{\sqrt{\cos^3 x\cos(x-\alpha)}}$ is equal to

(a) $2\sqrt{\tan x\sin\alpha + \cot\alpha} + c$

(b) $2\text{cosec}\alpha\sqrt{\tan x\sin\alpha + \cos\alpha} + c$

(c) $2\sqrt{\sin\alpha + \tan x\cos\alpha} + c$

(d) $2\text{cosec}\alpha\sqrt{\tan x\cos\alpha + \sin\alpha} + c$

10. $\int e^x \sin x\cos x\cos 2x\cos 4x\, dx$ is equal to

(a) $\frac{e^x}{4\sqrt{65}}\sin(8x - \tan^{-1}8) + c$

(b) $\frac{e^x}{8\sqrt{65}}\sin(8x - \tan^{-1}8) + c$

(c) $\frac{e^x}{8}\sin(8x - \tan^{-1}8) + c$

(d) None of these

11. $\int e^x\left(\frac{2\tan x}{1+\tan x} + \cot^2\left(x+\frac{\pi}{4}\right)\right)dx$ is equal to

(a) $e^x\tan\left(\frac{\pi}{4} - x\right) + c$ (b) $e^x\tan\left(x - \frac{\pi}{4}\right) + c$

(c) $e^x \tan\left(\frac{3\pi}{4} - x\right) + c$ (d) None of these

12. $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$ is equal to

(a) $2x\tan^{-1}(x) - \ln(x^2 + 1) + c$
(b) $2[\tan^{-1}(x) - \log_e(x^2 + 1)] + c$
(c) $2x\tan^{-1}(x) + \log_e(x^2 + 1) + c$
(d) None of these

13. $\int \frac{dx}{x + \sqrt{a^2 - x^2}}$ is equal to

(a) $\frac{1}{2}\sin^{-1}\frac{x}{a} + \ln\sqrt{x + \sqrt{a^2 - x^2}} + c$

(b) $\frac{1}{2}\sin^{-1}\frac{x}{a} + \ln(x\sqrt{a^2 - x^2}) + c$

(c) $\frac{1}{2}\sin^{-1}\frac{x}{a} + \ln(\sqrt{a + \sqrt{a^2 - x^2}}) + c$

(d) None of these

14. $I = \int \frac{\cos^4 x + 1}{\cot x - \tan x}dx$ is equal to

(a) $-\frac{1}{32}[\cos^2 2x + 4\cos 2x + 10\log_e \cos 2x] + c$

(b) $\frac{1}{32}[\cos^2 2x - 4\cos 2x + 10\log_e \cos 2x] + c$

(c) $\frac{1}{32}[\cos^2 2x - 4\cos 2x - 10\log_e \cos 2x] + c$

(d) None of these

15. $I = \int \tan^{-1}\left(\frac{1}{x^2 - x + 1}\right)dx$ is equal to

(a) $x\tan^{-1}\frac{1}{x^2 - x + 1} + \log_e \frac{x}{(2-x)\sqrt{1+x^2}\sqrt{2x - x^2}} + c$

(b) $x\cot^{-1}(x^2 - x + 1) + \log_e \frac{x}{(2-x)\sqrt{1+x^2}\sqrt{2x - x^2}} + c$

(c) $\tan^{-1}\left(\frac{1}{x^2 - x + 1}\right) + c$

(d) $x\tan^{-1}\frac{1}{x^2 - x + 1} - \tan^{-1}(1 - x)$
$+ \frac{1}{2}\log\frac{x^2 - 2x + 2}{x^2 + 1} + c$

16. $\int \frac{(x^2 - 1)}{(x^2 + 1)\sqrt{x^4 + 1}}dx$ is equal to

(a) $\sec^{-1}\left(\frac{x^2 + 1}{\sqrt{2}x}\right) + c$ (b) $\frac{1}{2}\sec^{-1}\left(\frac{x^2 + 1}{\sqrt{2}x}\right) + c$

(c) $\frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2 + 1}{\sqrt{2}}\right) + c$ (d) None of these

17. $I = \int \frac{(x + x^{2/3} + x^{1/6})}{x(1 + x^{1/3})}dx$ is equal to

(a) $\frac{3}{2}x^{2/3} + 6\tan^{-1}(x^{1/6}) + c$

(b) $\frac{3}{2}x^{2/3} - 6\tan^{-1}(x^{1/6}) + c$

(c) $\frac{3}{2}x^{2/3} + \tan^{-1}(x^{1/6}) + c$

(d) None of these

18. $\int\left(\frac{\ln x - 1}{(\ln x)^2 + 1}\right)^2 dx$ is equal to

(a) $\frac{x}{x^2 + 1} + c$ (b) $\frac{\ln x}{(\ln x)^2 + 1}$

(c) $\frac{x}{(\ln x)^2 + 1} + c$ (d) $e^x\left(\frac{x}{x^2 + 1}\right) + c$

19. If $\int \frac{dx}{(x - \sqrt{x^2 - 1})^2} = Ax^3 + Bx^2 + Cx$
$+ D(x^2 - 1)^{3/2} + E$, then

(a) $A = \frac{2}{3}, B = 0, C = -1$ (b) $A = \frac{1}{3}, B = 0, D = \frac{2}{3}$

(c) $A = D = \frac{2}{3}, B = 0, C = -2$

(d) None of these

20. $I = \int \frac{e^x}{(e^x + 1)^{1/4}}dx$ is equal to

(a) $\frac{e^{2x} - e^{-2x}}{2} + c$ (b) $\frac{e^{2x} + e^{-2x}}{2} + c$

(c) $\frac{e^x - e^{2x}}{2} + c$ (d) None of these

21. $I = \int \frac{\log_e(\log_e x)}{x(\log_e x)}dx$ is equal to

(a) $\frac{1}{2}\sqrt{\log_e(\log_e x)} + c$ (b) $\frac{1}{2}(\log_e(\log_e x))^2 + c$

(c) $\frac{1}{2}\log_e(\log_e x)$ (d) None of these

22. $I = \int \frac{(10x^9 + 10^x \log_e 10)}{(x^{10} + 10^x)}dx$ is equal to

(a) $10^x + x^{10} + c$ (b) $10^x - x^{10} + c$

(c) $10^x + x^{10}$ (d) $\log_e(10^x + x^{10}) + c$

23. $I=\int\sqrt{1+2\tan x(\sec x+\tan x)}\,dx$ is equal to

(a) $\log_e |\sec^2 x + \tan x \sec x| + c$

(b) $\log_e |1 - \tan x (\sec x + \tan x)| + c$

(c) $\log_e |\sin x (\sec x - \tan x)| + c$

(d) None of these

24. $\int\frac{x^{7/2}}{\sqrt{x^9+1}}dx$ is equal to

(a) $\sin^{-1}\sqrt{x^2+9}+c$ (b) $\frac{1}{2}\log\left(\frac{x^9+1}{x^9-1}\right)+c$

(c) $\frac{2}{9}\log\left(x^{9/2}+\sqrt{x^9+1}\right)+c$

(d) None of these

25. $\int\frac{dx}{(x^3-1)^{2/3}x^2}$ is equal to

(a) $\frac{3}{4}\left(1-\frac{1}{x^3}\right)^{4/3}+c$ (b) $(x^3 - 1)^{2/3} + c$

(c) $3\left(1+\frac{1}{x^3}\right)^{1/3}+c$ (d) $\left(1-\frac{1}{x^3}\right)^{1/3}+c$

26. $\int\left(\ln(1+\cos x)-x\tan\frac{x}{2}\right)dx$ is equal to

(a) $x \ln(1 + \cos x) + c$ (b) $x \ln(1 + \sec x) + c$

(c) $x^2 \ln(1 + \cos x) + c$ (d) $x \ln \tan x + c$

27. $\int e^x\left(\tan^{-1}x+\frac{2x}{(1+x^2)^2}\right)dx$ is equal to

(a) $e^x\left(\tan^{-1}x-\frac{1}{1+x^2}\right)+c$

(b) $e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)+c$

(c) $e^x\left(\cot^{-1}x-\frac{1}{1+x^2}\right)+c$ (d) None of these

28. The value of $\int\frac{x\tan x\sec x}{(\tan x-x)^2}dx$ is equal to

(a) $\frac{1}{\sin x-x\cos x}+c$ (b) $\frac{1}{x\cos x-\sin x}+c$

(c) $\frac{1}{x\sin x-\cos x}+c$ (d) None of these

29. $\int(x^2+x)(x^{-8}+2x^{-9})^{1/10}dx$ is equal to

(a) $\frac{5}{11}(x^2+2x)^{11/10}+c$ (b) $\frac{5}{6}(x+1)^{11/10}+c$

(c) $\frac{6}{7}(x+1)^{11/10}+c$ (d) None of these

30. If $I=\int x\log_e\left(1+\frac{1}{x}\right)dx=p(x)\ln\left(1+\frac{1}{x}\right)+\frac{1}{2}x$ $-\frac{1}{2}\ln(1+x)+c$, then

(a) $p(x)=\frac{1}{2}x^2$ (b) $p(x) = 0$

(c) $p(x) = 1$ (d) None of these

31. $\int\sqrt{\frac{x}{1+x^3}}dx$ is equal to

(a) $\frac{3}{2}\log_e(x+\sqrt{1+x^3})+c$

(b) $\frac{2}{3}\log_e(x^{3/2}+\sqrt{1+x^3})+c$

(c) $\frac{2}{3}\log_e(x^{3/2})\{1+\sqrt{x^{3/2}+x^{-3/2}}\}+c$

(d) None of these

32. $I=\int\frac{(\sin^8 x-\cos^8 x)}{1-2\sin^2 x\cos^2 x}dx$ is equal to

(a) $\sin 2x + c$ (b) $\frac{\sin 2x}{2}+c$

(c) $\frac{-\sin 2x}{2}+c$ (d) None of these

33. $I=\int\frac{\cos 2\theta}{(\sin\theta+\cos\theta)^2}d\theta$ is equal to

(a) $\frac{-1}{\sin\theta+\cos\theta}+c$ (b) $\log|\sin\theta + \cos\theta| + c$

(c) $\log|\sin\theta - \cos\theta| + c$ (d) $\log(\sin\theta + \cos\theta)^2 + c$

34. $\int e^{x^4}(x+x^3+2x^5)e^{x^2}dx$ is equal to

(a) $\frac{1}{2}xe^{x^2}e^{x^4}+c$ (b) $\frac{1}{2}x^2e^{x^4}+c$

(c) $\frac{1}{2}e^{x^2}e^{x^4}+c$ (d) $\frac{1}{2}x^2e^{x^2}e^{x^4}+c$

35. If $\int\frac{2dx}{((x-5)+(x-7))\sqrt{(x-5)(x-7)}}=f(g(x))+c$, then

(a) $f(x)=\sin^{-1}x, g(x)=\sqrt{(x-5)(x-7)}$

(b) $f(x) = \sin^{-1}x, g(x) = (x - 5)(x - 7)$

(c) $f(x)=\tan^{-1}x, g(x)=\sqrt{(x-5)(x-7)}$

(d) $f(x) = \tan^{-1}x, g(x) = (x - 5)(x - 7)$

SOLUTIONS

1. **(c) :** When $x \geq 0$: $\int x^n dx = \frac{|x|^n x}{n+1} + c$

When $x \leq 0$: $-\int x^n dx = \frac{(-x)^n x}{n+1} + c = \frac{|x^n| x}{n+1} + c$

2. **(d) :** Let $I = \int \frac{x^2 - 1}{x^3\sqrt{2x^4 - 2x^2 + 1}} = \int \frac{\left(\frac{1}{x^3} - \frac{1}{x^5}\right)dx}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}}$

Put $2 - \frac{2}{x^2} + \frac{1}{x^4} = z$

$\therefore\ I = \frac{1}{4}\int \frac{dz}{\sqrt{z}} = \frac{1}{2} \times \sqrt{z} + c\ \Rightarrow\ \frac{1}{2}\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$

3. **(b) :** $I_{m,n} = \int \cos^m x \cos nx dx$

$= \frac{\cos^m x \sin nx}{n} + \frac{m}{n}\int \cos^{m-1} x \sin x \sin nx dx$

$= \frac{\cos^m x \sin nx}{n}$

$\quad + \frac{m}{n}\int \cos^{m-1} x(\cos(n-1)x - \cos nx \cos x)dx$

$= \frac{\cos^m x \sin nx}{n} + \frac{m}{n}\int \cos^{m-1} x \cos(n-1)x dx - \frac{m}{n}I_{m,n}$

$\Rightarrow\ \frac{m+n}{n}I_{m,n} = \frac{\cos^m x \sin nx}{n} + \frac{m}{n}I_{m-1,n-1} + c_1$

or $(m + n)I_{m,n} = mI_{m-1,n-1} + \cos^m x \sin nx + c.$

4. **(d) :** $\int \frac{(1-x^7)}{x(1+x^7)}dx = P\log|x| + Q\log|x^7 + 1| + c$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{(1-x^7)}{x(1+x^7)} = \frac{P}{x} + 7Q\frac{x^6}{(x^7+1)}\ \Rightarrow\ P = 1, Q = -\frac{2}{7}$

5. **(c) :** $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$

$\Rightarrow\ \frac{e^{2x}}{2^2+5^2}f(x) + c = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + c$

$\Rightarrow\ a = 2, b = 5$ and $f(x) = 2\cos 5x + 5\sin 5x$

$\therefore$ Range of $f(x) = (-\sqrt{29}, \sqrt{29})$

6. **(b) :** Let $I = \int \frac{\cos x(1 + 4\cos 2x)}{\sin x + 4\sin x \cdot \cos^2 x}dx$

Put $\cos x = t\ \Rightarrow -\sin x dx = dt$

$\therefore\ I = -\int \frac{t[1 + 4(2t^2 - 1)]}{1 - t^2 + 4(1-t^2)(t^2)}dt = \int \frac{t[8t^2 - 3]}{4t^4 - 3t^2 - 1}dt$

$= \frac{1}{2}\ln|4t^4 - 3t^2 + 1| + c$

$= \frac{1}{2}\ln[|\sin x|] + \frac{1}{2}\ln|\sin x + 4\sin x\cos^2 x| + c$

7. **(b) :** $\int \frac{(x^2+1)}{(x^4 - x^2 + 1)\cot^{-1}\left(x - \frac{1}{x}\right)}dx$

$= \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x^2 + \frac{1}{x^2} - 2 + 1\right)\cot^{-1}\left(x - \frac{1}{x}\right)}dx = \int \frac{dt}{(t^2+1)\cot^{-1} t}$

$\left[\because\ \text{Put } x - \frac{1}{x} = t\ \Rightarrow\ \left(1 + \frac{1}{x^2}\right)dx = dt\right]$

$= \int \frac{-du}{u}\left[\because\ \text{Put } \cot^{-1} t = u \Rightarrow -\frac{1}{(1+t^2)}dt = du\right]$

$= -\ln|u| + c = -\ln\left|\cot^{-1}\left(x - \frac{1}{x}\right)\right| + c$

8. **(a) :** $I = \int \frac{dx}{\cos^3 x - \sin^3 x} = \int \frac{(\cos x - \sin x)dx}{(1 - \sin 2x)\left(1 + \frac{\sin 2x}{2}\right)}$

Put $\cos x + \sin x = t \Rightarrow (\cos x - \sin x)dx = dt$

$\therefore I = \int \frac{2dt}{(2-t^2)(t^2+1)} = \frac{2}{3}\int\left(\frac{1}{t^2+1} - \frac{1}{t^2-2}\right)dt$

$\quad = \frac{2}{3}\tan^{-1}(\sin x + \cos x) - \frac{1}{3\sqrt{2}}\ln(f(x)) + C$

9. **(b) :** Let $I = \int \frac{dx}{\sqrt{\cos^3 x \cos(x - \alpha)}}$

$= \int \frac{\sec^2 x dx}{\sqrt{\cos\alpha + \tan x \sin\alpha}}$

$= \int \frac{2tdt}{\sin\alpha \cdot t}$ [where $t^2 = \tan x \sin\alpha + \cos\alpha$]

$= 2\operatorname{cosec}\alpha . t = 2\operatorname{cosec}\alpha\sqrt{\tan x \sin\alpha + \cos\alpha} + c$

10. **(c) :** $\int e^x \sin x \cos x \cos 2x \cos 4x dx$

$= \frac{1}{2^3}\int e^x(2\sin x \cos x)(2\cos 2x)(2\cos 4x)\,dx$

$= \frac{1}{2^3}\int e^x(2\sin 2x \cos 2x)(2\cos 4x)dx = \frac{1}{2^3}\int e^x \sin 8x\,dx$

$= \frac{1}{2^3}\cdot\frac{1}{\sqrt{(1+8^2)}}e^x(\sin 8x - 8\cos 8x) + c$

$$= \frac{1}{8} \cdot \frac{1}{\sqrt{65}} e^x (\sin 8x - 8\cos 8x) + c$$

$$= \frac{e^x}{8} \sin(8x - \tan^{-1} 8) + c$$

11. (b) : $\int e^x \left(\frac{2\tan x}{1+\tan x} + \tan^2\left(x - \frac{\pi}{4}\right)\right) dx$

$$= \int e^x \left(\tan\left(x - \frac{\pi}{4}\right) + \sec^2\left(x - \frac{\pi}{4}\right)\right) dx$$

$$= e^x \tan\left(x - \frac{\pi}{4}\right) + c$$

12. (a)

13. (a) : Let $I = \int \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta$

(Put $x = a \sin\theta$, $dx = a \cos\theta \, d\theta$)

$$= \frac{1}{2}\int d\theta + \frac{1}{2}\int \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta} d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{2}\ln(\sin\theta + \cos\theta) + c$$

$$= \frac{1}{2}\sin^{-1}\frac{x}{a} + \ln\sqrt{x + \sqrt{a^2 - x^2}} + c$$

14. (a) : $I = \int \frac{\left[\left(\frac{1+\cos 2x}{2}\right)^2 + 1\right]\frac{1}{2}\sin 2x dx}{\cos 2x}$

Put $\cos 2x = p \Rightarrow -2\sin 2x \, dx = dp$

$$= -\frac{1}{4}\int \frac{\left[\left(\frac{1+p}{2}\right)^2 + 1\right]}{p} dp = -\frac{1}{16}\int \left(\frac{p^2}{p} + \frac{2p}{p} + \frac{5}{p}\right) dp$$

$$= -\frac{1}{16}\left[\frac{\cos^2 2x}{2} + 2\cos 2x + 5\ln(\cos 2x)\right] + c$$

15. (d) : Here, $f(x) = \int \tan^{-1}\left(\frac{(1-x)+x}{1-x(1-x)}\right) dx$

$$= \int (\tan^{-1}(1-x) + \tan^{-1} x) dx$$

$$= x\tan^{-1}(1-x) + \int \frac{x}{1+(1-x)^2} dx + x\tan^{-1} x - \int \frac{x}{1+x^2} dx$$

$$= x[\tan^{-1} x + \tan^{-1}(1-x)] - \ln\sqrt{1+x^2} + I_2,$$

where $I_2 = \int \frac{x}{2+x^2-2x} dx = -\int \frac{1-x-1}{2+x^2-2x} dx$

$$= -\int \frac{(1-x)}{1+(1-x)^2} dx + \int \frac{dx}{1+(1-x)^2}$$

$$= \frac{1}{2}\ln[1 + (1-x)^2 - \tan^{-1}(1-x)]$$

16. (b) : $I = \int \frac{x^2\left(1 - \frac{1}{x^2}\right) dx}{x^2\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)^{1/2}}$

Put $x + \frac{1}{x} = p \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dp$

$$\therefore I = \int \frac{dp}{p\sqrt{p^2 - 2}} = \frac{1}{2}\sec^{-1}\frac{p}{\sqrt{2}} = \frac{1}{2}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + c$$

17. (a) : Substituting $x = p^6$, $dx = 6p^5 \, dp$ in I, we have

$$I = \int \frac{6p^5(p^6 + p^4 + p)}{p^6(1+p^2)} dp = \int \frac{6(p^5 + p^3 + 1)}{(p^2+1)} dp$$

$$= \int 6p^3 \, dp + \int \left(\frac{6}{p^2+1}\right) dp = \frac{6p^4}{4} + 6\tan^{-1} p$$

$$= \frac{3}{2}x^{2/3} + 6\tan^{-1}(x^{1/6}) + c$$

18. (c) : Let $I = \int \left(\frac{\ln x - 1}{(\ln x)^2 + 1}\right)^2 dx$

Put $\ln x = t \Rightarrow \frac{dx}{x} = dt$

$$\therefore I = \int e^t \left(\frac{t-1}{t^2+1}\right)^2 dt = \int e^t \left(\frac{1}{t^2+1} - \frac{2t}{(t^2+1)^2}\right) dt$$

$$= \frac{e^t}{t^2+1} + c = \frac{x}{(\ln x)^2 + 1} + c$$

19. (a) : $\int \frac{dx}{\left(x - \sqrt{x^2-1}\right)^2} = \int \left(x + \sqrt{x^2-1}\right)^2 dx$

$$= \frac{2x^3}{3} + \frac{2}{3}(x^2-1)^{3/2} - x + c$$

20. (d) : Put $e^x = p \Rightarrow e^x \, dx = dp$

$$\therefore I = \int \frac{dp}{(1+p)^{1/4}} = \frac{4}{3}(1+p)^{3/4} + c = \frac{4}{3}(1+e^x)^{3/4} + c$$

21. (b)

22. (d) : Put $10^x + x^{10} = p \Rightarrow (10^x \log(10) + 10x^9) \, dx = dp$

$$\therefore I = \int \frac{dp}{p} = \log p = \log(x^{10} + 10^x) + c$$

23. (a) : $I = \int \sqrt{1 + 2\tan x(\sec x + \tan x)} \, dx$

$$= \int \sqrt{1 + 2\tan^2 x + 2\tan x \sec x} \, dx$$

$$= \int (\sec x + \tan x)\, dx = \log|\sec^2 x + \tan x \sec x| + c$$

24. (c) : Let $I = \int \frac{x^{7/2}}{\sqrt{x^9+1}}$ Put $x^{9/2} = t \Rightarrow \frac{9}{2}x^{7/2}dx = dt$

$$\therefore\ I = \frac{2}{9}\int \frac{dt}{\sqrt{t^2+1}} = \frac{2}{9}\log\left(t+\sqrt{t^2+1}\right)+c$$

$$= \frac{2}{9}\log\left(x^{9/2}+\sqrt{x^9+1}\right)+c$$

25. (d) : Let $I = \int \frac{dx}{(x^3-1)^{2/3}x^2} = \int \frac{dx}{\left(1-\frac{1}{x^3}\right)^{2/3}x^4}$

Put $1-\frac{1}{x^3} = t \Rightarrow \frac{3dx}{x^4} = dt$

$$\therefore\ I = \frac{1}{3}\int \frac{dt}{t^{2/3}} = t^{1/3} + c = \left(1-\frac{1}{x^3}\right)^{1/3} + c$$

26. (a) : $I = \int\left(\ln(1+\cos x) - x\tan\frac{x}{2}\right)dx$

$$= \int \ln(1+\cos x)\cdot 1\, dx - \int x\tan\frac{x}{2}dx$$

$$= \ln(1+\cos x)\cdot x + \int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}.x\, dx - \int x\tan\frac{x}{2}dx$$

$$= \ln(1+\cos x)\cdot x + \int x\tan\frac{x}{2}\ dx - \int x\tan\frac{x}{2}dx + c$$

$$= \ln\,(1+\cos x)\cdot x + c$$

27. (a) : $\int e^x\left(\tan^{-1}x + \frac{2x}{(1+x^2)^2}\right)dx$

$$= \int e^x\left(\tan^{-1}x + \frac{1}{1+x^2} - \frac{1}{1+x^2} + \frac{2x}{(1+x^2)^2}\right)dx$$

$$= e^x\left(\tan^{-1}x - \frac{1}{1+x^2}\right) + c$$

28. (b) : Let $I = \int \frac{x\tan x\sec x}{(\tan x - x)^2}dx$

$$= \int \frac{x\sin x}{\cos^2 x\frac{(\sin x - x\cos x)^2}{\cos^2 x}}dx = \int \frac{x\sin x}{(\sin x - x\cos x)^2}dx$$

Put $\sin x - x\cos x = t \Rightarrow\ x\sin x dx = dt$

$$\therefore\ \ I = \int \frac{dt}{t^2} = -\frac{1}{t} + c = \frac{1}{x\cos x - \sin x} + c$$

29. (a) : Let $I = \int (x^2+x)(x^{-8}+2x^{-9})^{1/10}dx$

$$= \int (x+1)(x^2+2x)^{1/10}dx$$

Put $x^2 + 2x = t\ \ \therefore\ I = \int t^{1/10}\cdot\frac{dt}{2}$

$$= \frac{1}{2}\times\frac{10}{11}t^{11/10} = \frac{5}{11}t^{11/10} + c = \frac{5}{11}(x^2+2x)^{11/10} + c$$

30. (a) : Here $I = \int x\log\left(1+\frac{1}{x}\right)dx$

$$= \frac{x^2}{2}\ln\left(1+\frac{1}{x}\right) - \int \frac{x^2.x}{2(1+x)}\left(-\frac{1}{x^2}\right)dx$$

$$= \frac{x^2}{2}\ln\left(1+\frac{1}{x}\right) + \frac{1}{2}\int dx - \frac{1}{2}\int\frac{dx}{1+x}$$

$$= \frac{x^2}{2}\ln\left(1+\frac{1}{x}\right)\ + \frac{1}{2}x - \frac{1}{2}\ln(1+x) + c$$

31. (b)

32. (c) : $I = \int \frac{\begin{matrix}(\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)\\(\sin^2 x - \cos^2 x)\end{matrix}}{1-2\sin^2 x\cos^2 x}dx$

$$= \int \frac{[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x](-\cos 2x)}{1-2\sin^2 x\cos^2 x}dx$$

$$= -\int \cos 2x\, dx = -\frac{\sin 2x}{2} + c$$

33. (b)

34. (d) : $I = \int e^{x^4}\ (x + x^3 + 2x^5)e^{x^2}$

Put $x^2 = t \Rightarrow 2xdx = dt\ \ \therefore\ \ I = \frac{1}{2}\int e^{t^2}(1+t+2t^2)e^t dt$

$$= \frac{1}{2}\int e^t[te^{t^2} + (e^{t^2} + 2t^2e^{t^2})]dt$$

$$= \frac{1}{2}\int e^t[f(t) + f'(t)]dt = \frac{1}{2}e^t(te^{t^2}) + c, \text{ where } t = x^2$$

35. (c) : $I = \int \frac{2dx}{2(x-6)\sqrt{(x-6)^2-1}}$. Put $(x-6)^2 - 1 = t^2$

$$\therefore\ I = \int \frac{dt}{t^2+1} = \tan^{-1}t + c = \tan^{-1}\left(\sqrt{(x-5)(x-7)}\right) + c$$

$$\Rightarrow\ f(x) = \tan^{-1}x \text{ and } g(x) = \sqrt{(x-5)(x-7)}$$

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Challenging PROBLEMS

1. The point (0, 5) is closest to the curve $x^2 = 2y$ at (x, y), then $\dfrac{x^2 + y}{2}$ is

(a) 1 (b) 2 (c) 6 (d) 4

2. If the sum of lengths of the hypotenuse and another side of a right angled triangle is given. The area of the triangle is maximum, then the angle between these is

(a) $\dfrac{\pi}{12}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

3. Let $x, y \in R$ in the interval (0, 1) and $x + y = 1$. The minimum value of the expression $x^x + y^y$ is k, then $[k]$ = ______ (where [.] denotes greatest integer function)

(a) 1 (b) 2 (c) 3 (d) 4

4. If r_1 and r_2 are the distances of points on the curve $10(Z\bar{Z}) - 3i(Z^2 - (\bar{Z})^2) - 16 = 0$ which are at maximum and minimum distance from the origin, then the value of $r_1 + r_2 =$

(a) 1 (b) 2 (c) 3 (d) 4

5. Locus of z if $\arg(z-(1+i)) = \begin{cases} \dfrac{3\pi}{4}, & \text{if } |z| \le |z-2| \\ -\dfrac{\pi}{4}, & \text{if } |z| > |z-4| \end{cases}$ is

(a) Straight lines passing through (2, 0)
(b) Straight lines passing through (2, 0), (1, 1)
(c) A line segment
(d) A set of two rays

6. Let $A(-4, 0)$ and $B(4, 0)$. If the number of points on circle $x^2 + y^2 = 16$ such that area of the triangle whose vertices are A, B and C is a positive integer, is N. Then the sum of digits in N is

(a) 4 (b) 6 (c) 8 (d) 10

7. If the curve $x^2 + 3y^2 = 9$ subtends an obtuse angle at the point $(2\alpha, \alpha)$, then a possible value of α^2 is

(a) 1 (b) 2 (c) 3 (d) 4

8. The length of the longest chord of the ellipse $4x^2 + 9y^2 = 1$ drawn through $\left(0, -\dfrac{1}{3}\right)$ is $\dfrac{3}{\sqrt{\lambda}}$, where λ is equal to ____

(a) 15 (b) 25 (c) 20 (d) 36

9. If n arithmetic means are inserted between two sets of numbers a, $2b$ and $2a$, b, where $a, b \in R$. Suppose that m^{th} arithmetic means between these two sets of numbers is same, then the ratio $a : b$ equals to

(a) $(n - m + 1) : m$ (b) $(n - m + 1) : n$
(c) $m : (n - m + 1)$ (d) $n : (n - m + 1)$

10. The sum of the series $1 + \dfrac{9}{4} + \dfrac{36}{9} + \dfrac{100}{16} + \ldots$ up to n terms, if $n = 16$ is

(a) 420 (b) 432 (c) 446 (d) 464

11. If 7 divides $32^{32^{32}}$, the remainder is

(a) 1 (b) 0 (c) 4 (d) 6

12. The family of curves passing through (0, 0) and satisfying the differential equation $\dfrac{y_2}{y_1} = 1$ (where $y_n = d^n y/dx^n$) is

(a) $y = k$ (b) $y = kx$
(c) $y = k(2^x + 1)$ (d) $y = k(e^x - 1)$

13. Two natural numbers a and b are selected at random. The probability that $a^2 + b^2$ is divisible by 7 is

(a) 3/8 (b) 1/7 (c) 3/49 (d) 1/49

14. The number of 4-digit numbers that can be formed by using the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 such that the least digit used is 4, when repetition of digits is allowed, is

(a) 617 (b) 671 (c) 716 (d) 761

15. The cartesian equation of the plane passing through the line of intersection of the planes $r \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and $r \cdot (\hat{i} - \hat{j}) + 4 = 0$ and perpendicular to the plane $r \cdot (2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$ is

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(a) $3x - 4y + 4z = 5$ (b) $x - 2y + 4z = 3$
(c) $5x - 2y - 12z + 47 = 0$
(d) $2x + 3y + 4 = 0$

16. In a three dimensional co-ordinate system P, Q and R are images of a point $A(a, b, c)$ in the xy, yz and zx planes respectively. If G is the centroid of triangle PQR, then area of triangle AOG is (O is the origin)
(a) 0 (b) $a^2 + b^2 + c^2$
(c) $\frac{2}{3}(a^2 + b^2 + c^2)$ (d) None of these

17. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set is
(a) decreased by 2
(b) two times the original median
(c) remains the same as that of the original set
(d) increased by 2

18. There exist positive integers A, B and C with no common factors greater than 1, such that
$A\log_{200}5 + B\log_{200}2 = C$. The sum $A + B + C$ equals to
(a) 5 (b) 6 (c) 7 (d) 8

19. In ΔABC, the bisector of the angle A meets the side BC at D and the circumscribed circle at E, then DE equals to
(a) $\frac{a^2\sec\frac{A}{2}}{2(b+c)}$ (b) $\frac{a^2\sin\frac{A}{2}}{2(b+c)}$
(c) $\frac{a^2\cos\frac{A}{2}}{2(b+c)}$ (d) $\frac{a^2\operatorname{cosec}\frac{A}{2}}{2(b+c)}$

20. In ΔABC, if $\cos A + \sin A - \frac{2}{\cos B + \sin B} = 0$, then $\frac{a+b}{c}$ is equal to
(a) $\sqrt{2}$ (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $2\sqrt{2}$

21. Number of common points for the curves $y = \sin^{-1}(2x) + \tan^{-1}\left(\frac{1}{[2x]}\right) + 2$ and $y = \cos^{-1}(2x + 5) + 1$ is (where [·] denotes greatest integer function)
(a) 0 (b) 1 (c) 3 (d) 4

22. If $I = \int_0^{\pi} x(\sin^2(\sin x) + \cos^2(\cos x))dx$, then $[I]$ = _____, (where [·] denotes the greatest integer function)
(a) 1 (b) 2 (c) 3 (d) 4

23. If A is a skew-symmetric matrix of order 3, then the matrix A^4 is
(a) skew symmetric (b) symmetric
(c) diagonal (d) none of these

24. The number of positive integral solutions of the equation $\begin{vmatrix} y^3+1 & y^2z & y^2x \\ yz^2 & z^3+1 & z^2x \\ yx^2 & x^2z & x^3+1 \end{vmatrix} = 11$ is
(a) 1 (b) 2 (c) 3 (d) 4

25. If $2\tan^2x - 5\sec x = 1$ for exactly 7 distinct values of $x \in \left[0, \frac{n\pi}{2}\right], n \in N$, then the greatest value of n is
(a) 13 (b) 17 (c) 19 (d) 15

26. The values of k for which lines $kx + 2y + 2 = 0$, $2x + ky + 3 = 0$, $3x + 3y + k = 0$ are concurrent is
(a) $\{2, 3, 5\}$ (b) $\{2, 3, -5\}$
(c) $\{3, -5\}$ (d) $\{-5\}$

27. The angle of elevation of a cloud from a point h mtrs. above the surface of a lake is θ and the angle of depression of its reflection in the lake is ϕ. The height of the cloud is
(a) $\frac{h\sin(\phi+\theta)}{\sin(\phi-\theta)}$ (b) $\frac{h\sin(\phi-\theta)}{\sin(\phi+\theta)}$
(c) $\frac{h\sin(\theta+\phi)}{\sin(\theta-\phi)}$ (d) $\frac{h\sin(\theta-\phi)}{\sin(\theta+\phi)}$

28. A function $f : R \rightarrow [1, \infty)$ satisfies the equation $f(xy) = f(x)f(y) - f(x) - f(y) + 2$. If f is differentiable on $R - \{0\}$ and $f(2) = 5$, $f'(x) = \frac{f(x)-1}{x}.\lambda$, then λ = ____
(a) $2f'(1)$ (b) $3f'(1)$
(c) $\frac{1}{2}f'(1)$ (d) $f'(1)$

29. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of distinct non parallel lines. If constant c is changed as a new constant k, then new equation represents
(a) pair of lines (b) parabola
(c) ellipse (d) hyperbola

30. From a point P outside a circle with centre at C, tangents PA and PB are drawn such that $\frac{1}{(CA)^2} + \frac{1}{(PA)^2} = \frac{1}{16}$, then the length of chord AB is
(a) 8 (b) 12 (c) 16 (d) 10

SOLUTIONS

1. **(c) :** $f(x) = x^2 + \left(\dfrac{x^2}{2} - 5\right)^2$

$$f'(x) = 2x + 2\left(\frac{x^2}{2} - 5\right)x = 2x\left(1 + \frac{x^2}{2} - 5\right)$$

$= x(x^2 - 8)$

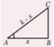
Now, $f'(x) = 0 \Rightarrow x^2 = 0, 8$

The point is closest for $x^2 = 8$.

2. **(c) :** $AB + AC = \text{constant} = k$

$\therefore \quad BC^2 = (k - x)^2 - x^2 = k^2 - 2kx$

$\therefore \quad \Delta = \dfrac{1}{2} BC \cdot AB = \dfrac{1}{2} x\sqrt{k^2 - 2kx}$



Let $Z = \Delta^2 = \dfrac{1}{4}x^2(k^2 - 2kx)$

$= \dfrac{1}{4}(k^2x^2 - 2kx^3)$

Z will be max, when $x = k/3$

Now, $\cos\theta = \dfrac{x}{k - x} = \dfrac{k/3}{k - k/3} = \dfrac{1}{2}$

$\therefore \quad \theta = \pi/3$

3. **(a) :** Let $f(x, y) = x^x + y^y \Rightarrow f(x) = x^x + (1 - x)^{1-x}$

$\Rightarrow \; f(x) = g(x) + g(1 - x)$

$\Rightarrow \; f'(x) = g'(x) - g'(1 - x)$

4. **(c) :** We have, $10(x^2 + y^2) - 3i(2x)(2iy) - 16 = 0$

$\Rightarrow \; 5(x^2 + y^2) + 6xy - 8 = 0 \qquad \text{...(i)}$

Let $(r\sin\theta, r\cos\theta)$ be the point on (i)

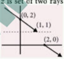
$\therefore \quad 5r^2 + 6r^2 \sin\theta\cos\theta - 8 = 0$

$\Rightarrow \quad r^2 = \dfrac{8}{5 + 3\sin 2\theta}$

$\therefore \quad r_1 + r_2 = 2 + 1 = 3$

5. **(d) :** $\text{Arg}\,(z - (1 + i)) = \begin{cases} \dfrac{3\pi}{4}, & x \le 1 \\ \dfrac{-\pi}{4}, & x > 2 \end{cases}$

$\therefore$ The locus of z is set of two rays



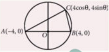
6. **(c) :** Required area of $\Delta ABC = \dfrac{1}{2} \cdot 8 \cdot 4\sin\theta = |16\sin\theta|$

Now area of Δ is a integer so the possible values of $\sin\theta$ are $\dfrac{1}{16}, \dfrac{2}{16}, \ldots\ldots, \dfrac{15}{16}$

i.e. 15 points in each quadrant

$\Rightarrow$ 60 + 2 more with $|\sin\theta| = 1$

$\Rightarrow \; N = 62$



7. **(b) :** The given curve is $\dfrac{x^2}{9} + \dfrac{y^2}{3} = 1$, whose director circle is $x^2 + y^2 = 12$. For the required condition $(2\alpha, \alpha)$ should lie inside the circle and outside the ellipse *i.e.*, $(2\alpha)^2 + 3\alpha^2 - 9 > 0$ and $(2\alpha)^2 + \alpha^2 - 12 < 0$

i.e., $\dfrac{9}{7} < \alpha^2 < \dfrac{12}{5}$.

8. **(c) :** Here $P = \left(\dfrac{1}{2}\cos t, \dfrac{1}{3}\sin t\right)$ and $Q = \left(0, -\dfrac{1}{3}\right)$

$\therefore \quad PQ^2 = -\dfrac{5}{36}\sin^2 t + \dfrac{2}{9}\sin t + \dfrac{13}{36}$

$(PQ^2)' = 0 \Rightarrow \sin t = \dfrac{4}{5}$

$\therefore \quad \text{Max of } (PQ^2) = \dfrac{9}{20} \Rightarrow \text{Max of } (PQ) = \dfrac{3}{\sqrt{20}}$

9. **(c) :** Let $A_1, A_2, \ldots\ldots, A_n$ be arithmetic means between a and $2b$, then $A_m = a + m\left(\dfrac{2b - a}{n + 1}\right)$

Again, let $B_1, B_2, \ldots\ldots B_n$ be arithmetic means between $2a$ and b, then $B_m = 2a + m\left(\dfrac{b - 2a}{n + 1}\right)$

Now, $A_m = B_m$ (Given)

$\Rightarrow a + m\left(\dfrac{2b - a}{n + 1}\right) = 2a + m\left(\dfrac{b - 2a}{n + 1}\right)$

$\Rightarrow m\left(\dfrac{b + a}{n + 1}\right) = a \Rightarrow \dfrac{a}{b} = \dfrac{m}{n - m + 1}$

10. **(c) :** The given series can be written as

$$1^3 + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \ldots$$

$$\therefore \quad t_n = \frac{1^3 + 2^3 + 3^3 + \ldots.. + n^3}{1 + 3 + 5 + \ldots. + (2n - 1)}$$

$$= \frac{n^2(n + 1)^2}{4n^2} = \frac{(n + 1)^2}{4}$$

$t_n = \dfrac{1}{4}(n + 1)(n + 1) = \dfrac{1}{4}(n^2 + 2n + 1)$

$\therefore \quad S_n = \dfrac{1}{4}\left[\dfrac{n(n + 1)(2n + 1)}{6} + n(n + 1) + n\right]$

$\therefore \quad S_{16} = \dfrac{1}{4}\left[\dfrac{16 \cdot 17 \cdot 33}{6} + 16 \cdot 17 + 16\right]$

$$= \frac{1}{4}[88 \times 17 + 272 + 16] = 446$$

11. (c) : $32 = 2^5 \Rightarrow (32)^{32} = (2^5)^{32}$
$= 2^{160} = (3 - 1)^{160} = 3m + 1, m \in N$
$\therefore \quad (32)^{32^{32}} = (32)^{3m+1} = 2^{5(3m+1)}$
$= 2^{3(5m+1)}2^2 = 4 \cdot 8^{5m+1} = 4(7 + 1)^{5m+1}$
$= 4(7n + 1), n \in N = 28n + 4$

$\therefore$ When 7 divides $(32)^{32^{32}}$, remainder = 4

12. (d) : $\frac{dp}{dx} = p\left(\text{where } p = \frac{dy}{dx}\right)$
$\Rightarrow \ln p = x + c \Rightarrow p = e^{x+c}$
$\Rightarrow \frac{dy}{dx} = ke^x \quad \Rightarrow y = ke^x + \lambda \qquad ...(i)$
$\because$ (i) satisfies (0, 0), so $\lambda = -k$
$\therefore \quad y = k(e^x - 1)$

13. (d) : a, b are of the form
$a, b \in \{7m, 7m+1, 7m+2, 7m+3, 7m+4, 7m+5, 7m+6\}$
Now, $a^2, b^2 \in \{7m_1, 7m_1 + 1, 7m_1 + 4, 7m_1 + 2, 7m_1 + 2, 7m_1 + 4, 7m_1 + 1\}$
For required condition, a^2, b^2 must be of the form $7m$.
$\therefore$ Required probability $= \frac{1}{49}$

14. (b) : Least digit used = 4
$\therefore$ We can use 4, 5, 6, 7, 8, 9. But remember that at least one 4 must be used.
Now, 1st blank can be filled in 6 ways.
2nd blank can be filled in 6 ways.
3rd blank can be filled in 6 ways.
4th blank can be filled in 6 ways.
$\therefore$ 4 blanks can be filled in 6^4 ways. But out of these, some may contain no digit 4 at all. Let us find them.
Each blank can be filled in 5 ways (by 5, 6, 7, 8, or 9)
$\therefore$ 4 blanks can be filled in 5^4 ways (no 4 at all)
$\therefore$ Required Number $= 6^4 - 5^4$ (at least one 4) = 671.

15. (c) : Equation of any plane passing through the intersection of the planes $r \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and $r \cdot (\hat{i} - \hat{j}) + 4 = 0$ is $2x - 3y + 4z - 1 + \lambda(x - y + 4) = 0$
or $(2 + \lambda)x - (3 + \lambda)y + 4z + 4\lambda - 1 = 0$
The plane is perpendicular to the plane
$r \cdot (2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$ if $2(2 + \lambda) + (3 + \lambda) + 4 = 0$
$\Rightarrow \quad 11 + 3\lambda = 0 \Rightarrow \lambda = -11/3.$
The required equation of the plane is
$3(2x - 3y + 4z - 1) - 11(x - y + 4) = 0$
$\Rightarrow \quad 5x - 2y - 12z + 47 = 0$

16. (a) : Point A is $(a, b, c) \Rightarrow$ Images of point A *i.e.*, P, Q and R are $(a, b, -c)$, $(-a, b, c)$ and $(a, -b, c)$ respectively.
$\therefore$ Centroid of triangle PQR is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$
$\Rightarrow \quad G \equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$
$\Rightarrow$ A, O, G are collinear $\therefore$ Area of triangle AOG is 0.

17. (c) : The median is the value of 5th item when items are arranged in increasing or decreasing order.
$\Rightarrow$ Median is unchanged

18. (b) : Given $A \log_{200} 5 + B \log_{200} 2 = C$
$\Rightarrow A \log 5 + B \log 2 = C \log 200 = C \log(5^2\,2^3)$
$= 2C \log 5 + 3\,C \log 2$
Hence, $A = 2C$ and $B = 3C$
for no common factor greater than 1, $C = 1$
So $A = 2$, $B = 3$. Thus $A + B + C = 6$

19. (a) : $AD \cdot DE = BD \cdot DC$

$$DE = \frac{BD \cdot DC}{AD} = \frac{\left(\frac{ac}{b+c}\right)\left(\frac{ab}{b+c}\right)}{\frac{2bc}{b+c} \cdot \cos\frac{A}{2}} = \frac{a^2}{2(b+c)}\sec\frac{A}{2}$$

20. (a) : Given $(\cos A + \sin A)(\cos B + \sin B) = 2$
$\Rightarrow \cos(A - B) + \sin(A + B) = 2$
$\Rightarrow \cos(A - B) = 1$ and $\sin(A + B) = 1$
$\Rightarrow A = B$ and $A + B = \frac{\pi}{2} \Rightarrow C = \frac{\pi}{2}$
$\therefore \quad \frac{a+b}{c} = \sqrt{2}$

21. (a) : As domain of first function is $\left[-\frac{1}{2}, 0\right) \cup \left\{\frac{1}{2}\right\}$ and domain of second function is $[-3, -2]$. So, there is no common point

22. (d) : $I = \int_0^{\pi} (\pi - x)(\sin^2(\sin x) + \cos^2(\cos x))dx$

$$\Rightarrow \quad 2I = 2\pi\int_0^{\frac{\pi}{2}} (\sin^2(\sin x) + \cos^2(\cos x))dx$$

$$\Rightarrow \quad I = \pi\int_0^{\frac{\pi}{2}} (\sin^2(\sin x) + \cos^2(\cos x)dx$$

MPP-9 CLASS XI ANSWER KEY

1.	(a)	2.	(a)	3.	(c)	4.	(b)	5.	(b)
6.	(d)	7.	(a,b)	8.	(a,b)	9.	(b,c)		
10.	(a,b,c,d)			11.	(a,b)	12.	(a,b,c,d)		
13.	(a,b,c)	14.	(a)	15.	(a)	16.	(b)	17.	(7873)
18.	(3)	19.	(2)	20.	(2)				

$$\Rightarrow \quad I = \pi\int_0^{\frac{\pi}{2}} (\sin^2(\cos x) + \cos^2(\sin x))dx$$

$$\Rightarrow \quad 2I = \pi\int_0^{\frac{\pi}{2}} 2dx \Rightarrow I = \frac{\pi^2}{2}$$

23. (b) : We have $A^T = -A$

Now, $(A^4)^T = (A \cdot A \cdot A \cdot A)^T = A^T A^T A^T A^T$

$= (-A)(-A)(-A)(-A)$

$= (-1)^4 A^4 = A^4$

24. (c) : Multiply R_1, R_2 and R_3 by y, z and x respectively and then take common y, z and x from C_1, C_2, C_3 respectively, then

$$\begin{vmatrix} y^3+1 & y^3 & y^3 \\ z^3 & z^3+1 & z^3 \\ x^3 & x^3 & x^3+1 \end{vmatrix} = 11 \Rightarrow \begin{vmatrix} 1 & 0 & y^3 \\ -1 & 1 & z^3 \\ 0 & -1 & x^3+1 \end{vmatrix} = 11$$

$(C_1 \to C_1 - C_2 \text{ and } C_2 \to C_2 - C_3)$

$\Rightarrow \quad 1(x^3 + 1 + z^3) + y^3(1) = 11 \Rightarrow x^3 + y^3 + z^3 = 10$

So, solutions are (1, 1, 2), (1, 2, 1) or (2, 1, 1)

25. (d) : Now, $\sec x = 3 \Rightarrow \cos x = \frac{1}{3}$

It gives two values of x in each of $[0, 2\pi]$, $(2\pi, 4\pi]$, $(4\pi, 6\pi]$ and one value in $6\pi + \frac{3\pi}{2} = 15\frac{\pi}{2}$

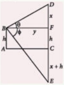
$\therefore$ Greatest value of $n = 15$

26. (c) : Three non parallel lines are concurrent if $\Delta = 0$

$$i.e., \quad \begin{vmatrix} k & 2 & 2 \\ 2 & k & 3 \\ 3 & 3 & k \end{vmatrix} = 0 \Rightarrow k = 2, 3, -5$$

But for $k = 2$, first two lines are parallel.

27. (a) : $\tan\theta = \frac{x}{y}$,



$\tan\phi = \frac{2h + x}{y}$

$\Rightarrow \quad x = \frac{2h}{\cot\theta \cdot \tan\phi - 1}$

$CD = h + x = \frac{h\sin(\phi + \theta)}{\sin(\phi - \theta)}$

28. (d) : $f'(x) = \lim\limits_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h\to 0} \frac{f\left\{x\left(1+\frac{h}{x}\right)\right\} - f(x)}{h} (x \neq 0 \text{ given})$

$= \lim\limits_{h\to 0} \frac{f\left(1+\frac{h}{x}\right) - 2}{\frac{h}{x}} \cdot \frac{f(x) - 1}{x}$

Putting $x = 1$, $y = 2$ in the given functional equation, we get $f(1) = 2$.

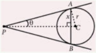
$\therefore \quad f'(x) = \frac{f(x) - 1}{x} \cdot \lim\limits_{h\to 0} \frac{f\left(1+\frac{h}{x}\right) - f(1)}{h/x}$

$= \frac{f(x) - 1}{x} \cdot f'(1)$

29. (d) : Since c is changed as k, $\Delta \neq 0$ and $h^2 > ab$.

$\therefore$ New equation represents hyperbola.

30. (a) : $\tan\theta = \frac{r}{PA}$



Given $\frac{1}{r^2} + \frac{1}{PA^2} = \frac{1}{16}$

$\Rightarrow \quad \frac{\cot^2\theta + 1}{(PA)^2} = \frac{1}{16}$

$\Rightarrow \quad PA\sin\theta = 4 = x \Rightarrow 2x = 8$

◆

SOLUTION SENDERS

Samurai Sudoku (December)

Dilip Raj	(Telangana)
Anoushka Patra	(West Bengal)
Neha Sam	(Kerala)

Mathdoku (December)

Anoushka Patra	(West Bengal)
Arvind Krishna	(Karnataka)
Rajul Jain	(U.P.)

Mathdoku (November)

Bhuvan Sun	(Tamil Nadu)

Maths Musing Set - 191

Harsh Kumar Choudhary	(Chattisgarh)
N. Jayanthi	(Hyderabad)

5 THINGS YOU NEED TO KNOW BEFORE CLASS 12TH MATHS EXAMINATION

Mathematics is one of the most scoring subjects. Though students practice well for their class 12th Maths paper, but many of them are averse to the subject. This may be due to mere fear for the subject or they are sometimes not confident enough. It is only through practice and time management that one can score high in their class 12th Maths examination. You are advised to practice from **MTG's Excel in Mathematics** for better preparation of your class 12th Maths Examination. You may also like to read our article on **how to boost your revision during Board exams**, please visit *blog.pcmbtoday.com*

Since Boards are nearing closer, and you will be occupied with exam preparation, there are chances that you might forget some important things to follow during your exams. Here, we are providing some important things that you should follow while attempting class 12th Maths Examination:

1. UTILISE THE FIRST 15 MINUTES

Questions on probability and the ones that are application-based are actually tricky. It is always advisable to read the question again-and-again to understand the context, as some key information might be hidden in the texts of the problem. So, as soon as you get the question paper, utilise the first 15 minutes to read and identify the probability and application-based questions, and underline the necessary keywords.

2. SOLVE THE KNOWN QUESTIONS WITH MAXIMUM MARKS FIRST

According to the latest sample papers released by CBSE, Class 12th Maths question paper is divided into four sections — Section A comprising 4 questions of one mark each, Section B comprising 8 questions of two marks each, Section C comprising 11 questions of 4 marks each and Section D comprising 6 questions of six marks each. Section D actually comprises 50% of the entire paper. You should attempt those questions first that carry more marks and the ones which you know how to solve. This will save your time and help you to prevent wasting time on repeated attempts in solving those questions you don't know. So, try attempting the questions of section D then try attempting questions of other sections as per the marking schemes.

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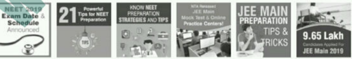
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KNOW NEET PREPARATION STRATEGIES AND TIPS
NTA Released JEE Main Mock Test & Online Practice Centers!
JEE MAIN PREPARATION TIPS & TRICKS
9.65 Lakh
Candidates Applied For JEE Main 2019

Indulge yourself in blog reading to keep yourself updated and ahead of time.

3. RE-CHECK YOUR ANSWERS WITH STANDARD FORMULAS AND EQUATIONS

In certain questions, the final answers can be checked and verified on the spot which can help to know if you have solved the questions correctly or not. For example, in the matrix method, by substituting values of x, y and z in any of the given equations, by evaluating the integral directly, the limit of sum can be checked. Also, by using the standard formulae, if vertices are given, students can check their answers in some AOI questions. So, check and verify your answers before submitting the answer sheet.

4. ATTEMPT ALL QUESTIONS

Since, there is no negative marking, you should try attempting all the questions. This will help you to score high in your class 12th Maths Exam paper. Moreover, each equation or formula carries some marks, writing the correct steps will earn you some marks, hence try your best to solve them even if you are unable to provide the correct answers. Writing the correct steps will earn you some marks. So, do not leave any question unanswered or you may lose marks unreasonably.

5. BE RELAX BEFORE THE EXAM

Exam creates nervousness in many students and they lose marks for answering wrong out of confusion. You are advised to keep calm and stay confident during your exams as you have prepared enough. Sleep well before your exam day and eat healthy. This will improve your concentration during the exam.

Now that you have just 3 months in hand to prepare for the Board exams, you should focus on important chapters from where most of the questions are asked in the Boards. As most of the questions asked in Boards are based on NCERT, you should solve all the **NCERT Textbook Exercise and Exemplar Problems** and practice all the solved examples given in NCERT textbook. Besides that, focusing on important topics will definitely help you to score high in your Maths paper. Here's a list of some important topics that you must not forget to revise before your exam:

1. Elementary transformation to find inverse of a square matrix **(NCERT textbook Exercise 4)**
2. Properties of determinants **(NCERT textbook Exercise 2)**
3. Matrix method **(NCERT textbook Exercise 6)**
4. Limit of sum **(NCERT textbook Exercise 8)**
5. Properties of definite integrals **(NCERT textbook Exercise 11)**
6. Application of integrals (chapter 8)
7. Concept of line and plane in 3-D (chapter 11)
8. Linear programming (chapter 12)
9. Baye's theorem **(NCERT textbook Exercise 3)**
10. Maxima/Minima application problems **(NCERT textbook Exercise 5)**

To make your Maths calculations easier and faster during class 12th Maths exam, learn some **Vedic Maths Tricks for Rapid calculations**. To score high in your class 12th Maths Exam Paper, you should practice as much as possible. Because in Maths, rigorous practice is the key for better preparation and accuracy. good result, hence to get good result. You must be very keen in solving problems which can be achieved only by solving **Sample Papers** and Practice **Previous Years' Papers**.

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JEE Mains January Session Sees Increase in Number of Female Applicants

The percentage of female applicants for JEE Main for January session has increased considerably this year. Despite the decrease in the total number of applicants appearing for the JEE Main exam in January, the percentage of female applicants has gone up to 29% as compared to 25% last year.

Approximately, 9.65 lakh students are appearing for the exam for January Session. However, for the previous session, the total numbers of applicants were close to 11.5 lakh which was held in April 2018. As many as 10.4 lakh students appeared in the examination, out of which only 24% were girls. Moreover, the JEE Main exam will be conducted by NTA from this year instead of CBSE which was the conducting body earlier. Also, the exam will be held twice a year.

In an attempt to restore gender diversity among engineering colleges in the country, government had issued a directive early this year. According to the directive, each of these institutes will have to admit at least 14% women by creating supernumerary seats. The plan is to achieve female enrolment of 14% in 2018-19, 17% in 2019-20, and 20% in 2020-21. NITs are also working towards increasing female enrolment in BTech programmes at the levels prescribed for IITs. Adding supernumerary seats in 2018 has resulted in creation of an extra 1,000 seats across all technical institutions.

A shift of the students is also being observed from the medical stream to engineering as the latter offers easy opportunities in respect to the students pursuing medical. The seats for medical is low and the expenses are also too much in the private colleges.

MATHS MUSING

SOLUTION SET-192

1. **(b)** : $S_9 = \frac{9}{2}(2a + 8d) = 9(a + 4d)$, $a_2 = 12 = a + d$

so, $S_9 = 27(16 - a)$.

According to question, $200 < S_9 < 220$

$\Rightarrow 200 < 27(16 - a) < 220 \Rightarrow 7.85 < a < 8.59$

So, $a = 8$, only possible integer and $d = 4$

2. **(c)** : $\lim\limits_{x\to 0} \frac{x - e^x + 1 - (1 - \cos 2x)}{x^2}$

$$= \lim_{x\to 0}\left(\frac{(x+1)-(1+x+x^2/2+\ldots)}{x^2} - \frac{2\sin^2 x}{x^2}\right) = -\frac{5}{2}$$

3. **(d)** : We will consider the following cases,

Case I : 4 alike and 2 other alike

Case II : 4 alike and 2 different

Case III : 3 alike and 3 other alike

Case IV : 3 alike and 2 other alike and 1 different

Total ways

$$= 1\times\frac{6!}{4!\times 2!} + 1\times\frac{6!}{4!} + 1\times\frac{6!}{3!\times 3!} + {}^2C_1\times\frac{6!}{3!\times 2!}$$

$= 15 + 30 + 20 + 120 = 185.$

4. **(b)** : Here, $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$

$$= \left\{\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1} - \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)}\right\}^{10}$$

$= (x^{1/3} - x^{-1/2})^{10}$ $\therefore$ $T_{r+1} = (-1)^r\,{}^{10}C_r\, x^{\frac{20-5r}{6}}$

For term to be independent of x, $\frac{20-5r}{6} = 0 \Rightarrow r = 4$

$\therefore$ Term $= {}^{10}C_4 = 210.$

5. **(b)** : We have, $S_k = \int \sin x\, d(i^k x) = \int i^k \sin x\, dx.$

Now, $\sum\limits_{k=1}^{4n-1} i^k = -1$

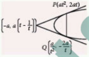
$\therefore$ $\sum\limits_{k=1}^{4n-1} S_k = -\int \sin x\, dx = \cos x + c$

6. **(b, c)** : Given equation is

$$\frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\sin x} + \frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\cos x} = 2 \Rightarrow \sin\left(\frac{\pi}{12} + x\right) = \sin 2x$$

$\Rightarrow x = \frac{\pi}{12}$ or $\frac{11\pi}{36}$

7. **(b)** : Given $a\left(t - \frac{1}{t}\right) = -a.$ $\therefore$ $t - \frac{1}{t} = -1$


$P(at^2, 2at)$
$\left[-a, a\left[t - \frac{1}{t}\right]\right]$
$Q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

$$PQ = a\left(t + \frac{1}{t}\right)^2 = a\left\{\left(t - \frac{1}{t}\right)^2 + 4\right\} = 5a$$

8. **(d)** : As $t - \frac{1}{t} = -1$, we have $t + \frac{1}{t} = \sqrt{5}$

The angle θ between them

$$\tan\theta = \frac{\frac{2}{t} + 2t}{1-4} = \frac{2\left(t + \frac{1}{t}\right)}{-3} = -\frac{2\sqrt{5}}{3}$$

9. **(6)** : $\frac{dy}{dx} = x\frac{dp}{dx} + p + \frac{dp}{dx} - 3p^2\frac{dp}{dx}$

$\Rightarrow p = x\frac{dp}{dx} + p + \frac{dp}{dx} - 3p^2\frac{dp}{dx}$

$\Rightarrow \frac{dp}{dx}(x + 1 - 3p^2) = 0 \Rightarrow \frac{dp}{dx} = 0$ or $p = \pm\sqrt{\frac{x+1}{3}}$

$\therefore$ $y = \pm\sqrt{\frac{x+1}{3}}\left(x + 1 - \frac{x+1}{3}\right)$

$\Rightarrow y^2 = \frac{4(x+1)^3}{27}$ $\therefore$ $k = \frac{27}{4} \Rightarrow [k] = 6$

10. **(c)** : **(P) → 2, (Q) → 4, (R) → 3, (S) → 1**

(P) $A^2 = -I.$ $\therefore$ A is of even order

(Q) $(I + A)^n = {}^nC_0 I + {}^nC_1 IA + {}^nC_2 IA^2 + \ldots + {}^nC_n A^n$
$= {}^nC_0 I + {}^nC_1 A + {}^nC_2 A + \ldots + {}^nC_n A = I + (2^n - 1)A$

(R) Given, $A^2 = A$ and $B = I - A$
$\therefore AB + BA + I - (I + A^2 - 2A) = AB + BA - A + 2A$
$= AB + BA + A = A(I - A) + (I - A)A + A = A$

(S) Given, $A^* = A$ and $B^* = B$
$\therefore (AB - BA)^* = B^*A^* - A^*B^* = BA - AB$

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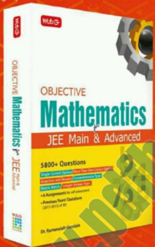
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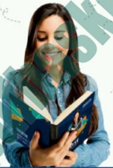

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